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ON THE ISSUE OF RELIABILITY CALCULATION FOR REDUNDANT STRUCTURES IN VIEW OF AGEING ELEMENTS

This paper considers the issue related to calculation of reliability parameters of redundant structures in view of ageing elements. Maintenance of facilities at modern industrial enterprises shows that basic parts as well as spare parts are subject to ageing during their functioning. Failed items are subject to restoration, and after repair they are added to the stock of spares. However, one should take into account that repairing generally brings only partial recovery of functionality. An item uses up a part of its life time during the previous operation, and during repair full recovery is not achieved. The problem of defining the structure of spares, with incomplete (partial) recovery of redundant elements and depletion of some part of their life time taken into account, is solved by simulation modelling.

Keywords: *safety, reliability, spares, redundancy, geometrical process, life time, transition graph, incomplete recovery.*

Introduction

For maintenance of industrial facilities, in particular facilities with increased risk, such as nuclear power installations (NPI), very high requirements are assigned to their safety and reliability. One of the ways to increase the level of reliability is to plan preventive maintenance, to control availability of functioning objects, and to build sets of spares for operative replacement of failed equipment.

In the present paper we shall consider issues related to calculating reliability of repairable equipment, with available spares taken into account, and defining required amounts of spares to insure specified reliability parameters.

Issues related to calculating reliability of equipment in view of spares and defining their optimal structure were considered in the works of Russian and foreign experts. The review of the literature on the issue can be found, for example, in [1, 2]. Here, it is worth mentioning that issues of calculation and optimization of spares are presented in corresponding handbooks, references to which can also be found in studies [1, 2], and they are reflected in GOST (see references, for example, in [3]). However, it should be noted that the issue has not lost its importance nowadays and is still at the focus of experts. Thus, in the *Dependability* journal there is an ongoing discussion about updating the method of spares estimation presented in the corresponding GOST [4] and the guidelines [5].

Note one feature characteristic of much of the equipment in operation in various industries. The feature consists in the fact that the equipment has a great mean time to failure. Very often this is higher than the resource or life time defined in specifications. In many industries there are facilities still functioning that were put into operation during the Soviet time. Refurbishment is being carried out slowly. Consequently, we can expect that in these facilities there are processes of ageing caused by wear of materials and degradation processes occurring inside of products. In reference with this, there arises an issue of calculating the reliability of equipment with available spares in view of ageing.

1. Functioning process of restorable maintainable products

Let us consider a method for calculating of the reliability characteristics of a restorable and maintainable item with *n* spares available. The functioning strategy of an item is as follows. At the initial moment of time the item is in good state. The item fails with the rate $\lambda(t)$. In case of failure, the item is replaced by a spare one. The rate of item replacement is equal to $\mu(t)$. The faulty item is sent for repair. After repair the item is considered as one with restored availability and is transferred into the reserve. Denote a repair rate as $\nu(t)$. If there is no more serviceable item in the reserve, a failure occurs. The described strategy of functioning can be presented with the help of the graph, as is shown in Figure 1.

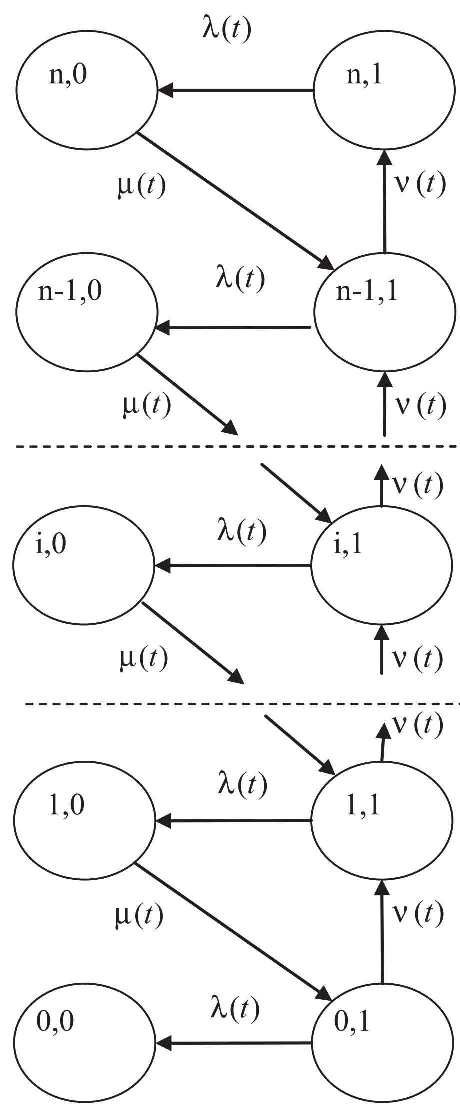

Fig. 1. Graph of transitions of a restorable maintainable item

It is necessary to estimate system reliability characteristics (availability factor, probability of a system's failure due to the absence of spare items) for the considered strategy and to determine a required quantity of spares to ensure the set level of system availability.

The state of an object on the graph will be denoted by two symbols (k, i) where the first symbol means the quantity of spares, $k=0$, …, n, and the second symbol i means the state of a basic item under load, $i=1$ means that the item is available, $i=0$ means that the item is unavailable.

Let us examine the functioning of an item with spares available in more detail. In the beginning of operation the item is in state (n, 1) with probability 1 (there are n spares available, the item is sound). At a random moment of time, with a failure rate $\lambda(t)$, the item transits in state (n, 0) (n spares, the item in failure state, and replacement of the item begins). With a repair rate $\mu(t)$ the item transits in state $(n-1,1)$ ($n-1$ spares, the item is sound). From this state the item can transit in state (n, 1) with a repair rate $v(t)$ (repair is finished, and there are again n spares in the reserve), or in state $(n-1,0)$ with a rate $\lambda(t)$ (repair is not finished before a next failure), and so on. When getting into state (0,0), the item stops functioning and is in a failure state till the moment when spares are replenished.

In this problem the interest is in calculating the probability of being in each of intermediate states $P_{i,j}(t)$, where $i = \overline{1,n}, j = 0,1$. Special interest is presented by the problem of defining the probability of getting into state $P_{0,0}(t)$ characterized by failure of a working item and absence of spares.

The considered strategy of functioning can be described by a non-stationary Markov process and represented as a system of differential equations

$$
\begin{array}{l}
dP_{n,1}(t)/dt = -\lambda(t)P_{n,1}(t) + \nu(t)P_{n-1,1}(t) \\
dP_{n,0}(t)/dt = -\mu(t)P_{n,0}(t) + \lambda(t)P_{n,1}(t) \\
-------------------- \\
dP_{i,1}(t)/dt = \mu(t)P_{i+1,0}(t) + \nu(t)P_{i-1,1}(t) - (\lambda(t)+\nu(t))P_{i,1}(t) \\
dP_{i,0}(t)/dt = \lambda(t)P_{i,1}(t) - \mu(t)P_{i,0}(t) \\
-------------------- \\
dP_{0,1}(t)/dt = \mu(t)P_{1,0}(t) - (\lambda(t)+\nu(t))P_{0,1}(t) \\
dP_{0,0}(t)/dt = \lambda(t)P_{0,1}(t)
\end{array} \qquad (1)
$$

However, the analytical solution of the given problem presents significant difficulties. In the given paper we shall solve this task by methods of simulation modelling.

To describe the relationship of failure rates of items, we shall use the model of geometrical process.

2. Geometrical processes

Take the model applied to describe the change of reliability parameters of facilities that takes into account ageing of facilities while in service or incomplete restoration of functionality after failure [6].

Consider the following strategy of functioning of a facility. The item functions well for random time. It is recovered after failure. The recovery is assumed to be in complete. The incompleteness of restoration is characterized by the degradation factor γ. Suppose that recovery of an item takes a negligibly little time compared to mean times between failures (practically instantly). As a result of incomplete restoration, the MTBF ξ of the restored object is reduced (in terms of probability) in γ times in comparison with the previous operation phase:

$$\xi_2 \overset{d}{=} \gamma\ \xi_1, \ldots \xi_n \overset{d}{=} \gamma^{n-1}\xi_1, 0 < \gamma \le 1.$$

In other words, we can say that the MTBF of the restored item for an observed time interval in relation to a new object will decrease in terms of probability proportional to some coefficient. Mathematically, the relationship between functions of distribution of mean times between failures of a restored object with incomplete restoration taken into account can be written

$$F_{\xi_2}(t) = F_{\xi_1}\left(\frac{t}{\gamma}\right), \ldots, F_{\xi_n}(t) = F_{\xi_1}\left(\frac{t}{\gamma^{n-1}}\right),$$

where $F_{\xi_i}(t)$ is the function of distribution of mean time between failures of a *(i*-1*)*-time restored object, γ is the factor of restoration incompleteness, factor of degradation or wear. Corresponding densities of distribution are related by

$$f_{\xi_n}(t) = \frac{1}{\gamma^{n-1}} f_{\xi_1}\left(\frac{t}{\gamma^{n-1}}\right).$$

The degradation factor means the following: γ is the averaged quantity reflecting the process of accumulation of damages, defects, indirectly describing the process of gradual fatigue of material, process of physical ageing, wearability, embrittlement, corrosion, etc. In some cases γ can be understood as the factor reflecting increase of load imposed on the object due to variable modes of operation.

Definition. Let $\{\xi_i\}, i \ge 1$ be the sequence of independent random variables representing mean times between failures of an object with the corresponding distribution functions $F_{\xi_i}(t)$ generated by the distribution $F(t)$, then

$$F_{\xi_i}(t) = F\left(\frac{t}{\gamma^{i-1}}\right),\ i=1,2,\ldots,$$

where γ is a positive constant. Then the sequence $\{\xi_i\}, i \ge 1$ is defined as geometrical process.

Let us use the expression relating the failure rate at the initial stage of functioning and that after *(n-1)*-th failure. By definition, the failure rate is equal to the ratio

$$\lambda(t) = \frac{f(t)}{1 - F(t)}.$$

Write down the expression for failure rate of the object restored *(n-1)* times

$$\lambda_{\xi_n}(t) = \frac{f_{\xi_n}(t)}{1 - F_{\xi_n}(t)} = \frac{\frac{1}{\gamma^{n-1}} f_{\xi_1}\left(\frac{t}{\gamma^{n-1}}\right)}{1 - F_{\xi_1}\left(\frac{t}{\gamma^{n-1}}\right)} = \frac{1}{\gamma^{n-1}} \lambda_{\xi_1}\left(\frac{t}{\gamma^{n-1}}\right). \quad (2)$$

Thus, a failure rate after each restoration becomes in $\frac{1}{\gamma}$ times higher than the failure rate on the previous time interval, with the time scale on which the process is defined changing as well.

3. Task solution

Modeling process is arranged according to the description presented in Section 1. At the initial moment of time an object is in good state. Then, at a random moment of time the object fails. The failed item is removed from operation, sent to repair, and an item from spares is put in its place. After repair the object is added back to the stock of spares, being placed at the end of the use queue. That is, the next time it will be installed into the system only after all spares ahead of it at the queue have worked to failure. The same applies to process of installation, failure, repair and returning into the stock of spares for all other items. It should be noted that during the first cycle of use, facilities have a failure rate $\lambda_{\xi_1}(t)$, after the first failure the rate changes in consistence with Expression (2) and is equal to $\lambda_{\xi_2}(t)$, and after the *i-th* failure, it will have the value $\lambda_{\xi_{i+1}}(t)$.

Now, consider the results of test calculations carried out as to the described technique. As initial data for calculation, the following values were used:

- Item failure rate $\lambda(t)$ =0.001 1/hour,
- Replacement rate of an item $\mu(t)$ =1 1/hour,
- Repair rate $\nu(t)$ =) = 0,1 1/hour. The number of realizations of simulation modeling is *N=105*. The operating time of a redundant structure is assumed to be equal to 16000 hours.

At the first stage we shall calculate the changes of an item's average operating time depending on the amount of restorations. In parallel, we shall change the degradation factor. For calculations, the structure with one basic item and three spares was considered. Figure 3 shows the diagrams of the behavior of an item's mean time to failure depending on the time of functioning of the structure for various values of degradation factor. WT = operating time, RC = number of restorations, α = degradation factor.

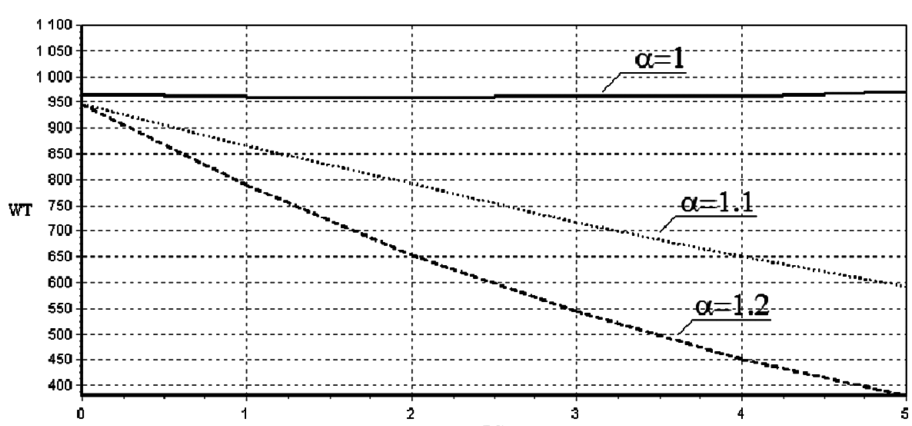

Fig. 2. Change of mean time between failures depending on the restoration number

The presented diagrams clearly demonstrate that the mean time to failure does not change and is equal to 1000 hour for items, without ageing taken into account. For all other diagrams, the situation is that the less the degradation factor is, the more quickly the mean time to failure decreases. For the given example with the set values of parameters, the availability factor has been calculated. The results are presented in Table 1.

Table 1

Value of parameter γ	1	1/1,05	1/1	1/1,15	1/1,2
Availability factor	0,999	0,9989	0,9988	0,9986	0,9979

The results of calculations show that the availability factor decreases as the degradation factor increases.

The offered mathematical model can be used for solving the task related to calculating a required amount of spares to guarantee the compliance with specified reliability parameters on a considered time interval of a structure's functioning. Let us set an availability factor as the reliability parameter. Let it be required to ensure the value of the availability factor as not less than 0.99 during 16000 hours of operation. The results of the calculations are presented in Table 2.

Table 2

Value of parameter γ	1	1/1,05	1/1,1	1/1,15	1/1,2
Number of spares ensuring specified requirements	2	2	2	2	3
Availability factor	0,9964	0,9974	0,9943	0,9928	0,9979

As is shown in Table 2, the specified requirements for an availability factor are satisfied by a structure with one basic and two spares for degradation factors $\gamma = 1 \div 1/1,15$, and with three spares for $\gamma = 1/1,2$.

Conclusion

Thus, we can say that this paper presents a model of calculating the reliability of a redundant structure, with ageing of items taken into account. The assumption has been made that during the functioning of a system, a basic element as well as spares use up a part of their service life. As a result, a partial restoration of system functionality takes place. The behavior of system parameters depending on the value of degradation factor has been studied using test examples. It has been demonstrated that the developed model can be used for calculating a required number of spares to guarantee the compliance with specified reliability parameters on a considered time interval of a structure's functioning.

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