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ESTIMATION OF RELIABILITY INDICES OF A “LINEARLY AGEING” OBJECT

This paper describes methodological difficulties when dealing with practical engineering challenges of reliability for non-stationary (time dependent/“ageing”) objects. A special case is considered when the object’s failure rate is linearly growing with operation time. An average lifetime of such object is defined. The result expression is reduced to the formula which is accessible to be used in engineering analyses. A formal way of substitution of a real non-stationary “ageing” object for a virtual stationary one is proposed. Besides, a constant failure rate of a virtual object is taken on the basis of additional considerations, in particular, on the basis of the condition ensuring the equality of the “life times” of both objects. The formulas are developed for calculation of the failure rate for a virtual stationary object, expressed through the parameters of the real object’s “ageing” characteristics. The efficiency of the suggested method is demonstrated by means of the solved problem about final probabilities of the states of the “linearly ageing” object and its availability factor, deduced in analytical form.

Keywords: *reliability; non-stationary object; failure rate; object’s lifetime; stationarization; final probabilities of states; availability factor.*

The reliability theory is widely used in the engineering practice to solve many important tasks, such as, for instance, the assignment or extension of operational life of an object (element, system), development of the scientifically based methods of its deployment, determination of inspection frequency, preventive and overhaul repairs and other measures to ensure its specified reliability level during operation process, etc. Successful handling of such problems requires the knowledge of reliability characteristics of the object, deduced in analytical form.

These are the three main reliability characteristics that are mostly used in engineering analyses: failure rate $\lambda(t)$, time-to-failure density function $f(t)$ and reliability function $p(t)$ – probability of non-failure operation during the time period t. Generally, all these characteristics are the functions of time which are interrelated. This means that the knowledge of one of them gives the opportunity to define any of the remaining ones by the known formulas [1].

Under operational conditions these characteristics are deduced by means of acquisition and processing of the statistics data on the objects in operation. Generally, the most accessible is to reveal the function $\lambda(t)$, as, for any important objects, the failures and attendant circumstances are normally documented by operating personnel, and the information received is available for further analysis. Then, the characteristics $\lambda(t)$, $f(t)$ and $p(t)$ are known in analytical form and can be used to solve the research tasks and practical estimations.

The vast majority of the present engineering methods of reliability estimations is based on the hypothesis of stability of the flow of random events. It means that all process probabilistic characteristics remain unchanged with time, for instance, $\lambda(t)$=const=λ_0, and it leads to the known exponential correlations:

$$f(t) = \lambda_0 \cdot e^{-\lambda_0 t};\ p(t) = e^{-\lambda_0 t}.$$

The hypothesis of stability of reliability-related processes within a long-term time interval of an object's functioning in many cases is quite convincing [2]. But it is actually a fact that as time goes by, the certain real objects obviously tend to the failure rate rise, i.e. they are non-stationary. In the reliability theory such objects are usually called ageing objects [1].

At present there are almost no admitted engineering methods of calculation of the ageing/deteriorating objects reliability indicators, although the algorithm of their determination remains the same as in case of stability. Ultimately, the attempts of application of this algorithm for analytical determination of major reliability characteristics of a non-stationary object in two or three steps, generally, cause significant mathematical difficulties. These difficulties may appear when the noted differential equations (for instance, the Kolmogorov equations [1]) are not solved in quadratures, or when some integrals established as the result of mathematical calculations are not expressed in elementary terms and can be defined only numerically. Consequently, it is almost impossible to find the required calculation formulas in the reference materials, or they are quite complicated and, in addition, are derived with simplifying assumptions.

Some practical reliability tasks basically could be solved with the use of numerical characteristics of a random variable (time to failure): one of its so called moments is a mean time to failure (an object's lifetime) T. And all the aboveindicated difficulties related to the determination of T for a non-stationary object remain in force. There are cases, however, when this task can be solved completely. One of such cases is the situation when variation of the failure rate of an object becomes linear as time goes by ("linearly ageing object"). The solution of this task is given below.

So let us assume that $\lambda(t)$ is described by the following function:

$$\lambda(t) = \lambda_0 + at, \tag{1}$$

where λ_0 is an initial failure rate, a is a coefficient of an object's ageing ($a \geq 0$), t is current time. Then λ_0 and a are considered to be known and assigned.

Let us find the reliability function of such object $p(t)$, which is associated [3] with $\lambda(t)$ by the correlation:

$$p(t) = e^{-\int_0^t \lambda(t)dt}, \tag{2}$$

with consideration of (1) it results in:

$$p(t) = e^{-\left(\frac{\alpha}{2}t^2 + \lambda_0 t\right)}. \tag{3}$$

By means of simple transformations the index of this constituent is developed to the form:

$$-\left(\frac{\alpha}{2}t^2 + \lambda_0 t\right) = -\left(\sqrt{\frac{\alpha}{2}}t + \frac{\lambda_0}{\sqrt{2\alpha}}\right)^2 + \frac{\lambda_0^2}{2\alpha},$$

and thus (3) can be written over to:

$$p(t) = e^{-\left(\sqrt{\frac{\alpha}{2}}t + \frac{\lambda_0}{\sqrt{2\alpha}}\right)^2} \cdot e^{\frac{\lambda_0^2}{2\alpha}} \tag{4}$$

Let us define T as a mean time to failure for such object. It is known [1, 3] that $T = \int_0^\infty p(t)dt$; consequently, for the current case:

$$T = e^{\frac{\lambda_0^2}{2\alpha}} \cdot \int_0^\infty e^{-\left(\sqrt{\frac{\alpha}{2}}t + \frac{\lambda_0}{\sqrt{2\alpha}}\right)^2} dt. \tag{5}$$

For calculation of this integral let us change the variables: $\sqrt{\frac{\alpha}{2}}t + \frac{\lambda_0}{\sqrt{2\alpha}} = z$. Then $dt = \sqrt{\frac{2}{\alpha}}dz$ and in consideration with the fact that with $t = 0$ $z = z_0 = \frac{\lambda_0}{\sqrt{2\alpha}}$, the formula (5) is developed to the form:

$$T = \sqrt{\frac{2}{\alpha}} \cdot e^{\frac{\lambda_0^2}{2\alpha}} \cdot \int_{\frac{\lambda_0}{\sqrt{2\alpha}}}^{\infty} e^{-z^2} dz = \sqrt{\frac{2}{\alpha}} \cdot e^{\frac{\lambda_0^2}{2\alpha}} \cdot \frac{\sqrt{\pi}}{2} \cdot \left[1 - \mathrm{F}\left(\frac{\lambda_0}{\sqrt{2\alpha}}\right)\right], \tag{6}$$

where $\mathrm{F}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2} dt$ is the probability integral, for calculation of which the detailed tables [4] are used. Hence, the task when it is necessary to find a mean lifetime of a "linearly ageing/deteriorating" object is solved analytically.

Determination of T by the formula (6) is often inconvenient, as it is associated with the necessity of calculation of the difference between two small close numbers to high precision. This difficulty can be circumvented by the following way.

Let us limit further consideration by the case when $z_0 = \frac{\lambda_0}{\sqrt{2\alpha}} > 1$. The failure rate λ_0 within a fixed time period T_f of reliability assessment of an ageing/deteriorating object has increased by the amount of β (β>1). Then $\lambda(t{=}T_f) = \beta\lambda_0$. And according to (1) it follows that $\beta\lambda_0 = \lambda_0 + aT_f$ and $\alpha = \frac{(\beta - 1)\lambda_0}{T_f}$. The statement z_0>1, after it is substituted with these values of a, transforms into:

$$\lambda_0 T_f > (\beta - 1),$$

i.e. the condition z_0>1 means that within the fixed interval of reliability assessment the increase of the average number of the object's failures due to ageing does not exceed the average number of failures at the initial stage of its functioning. Physically it means that the situation is being analyzed when the object's ageing with time goes on relatively slow; if the object is ageing/deteriorating too fast, it hardly makes practical sense speaking about any of its long-term operation.

Let us put to use that for $z_0>1$, the following asymptotic decomposition is valid [4]:

$$\frac{\sqrt{\pi}}{2}\left[1-\mathrm{F}(z)\right]\approx\frac{e^{-z^2}}{2z}\left[1-\frac{1}{2z^2}+\frac{1\cdot 3}{(2z^2)^2}-\frac{1\cdot 3\cdot 5}{(2z^2)^3}+...\right],$$

and the error occurring when the row is limited, in absolute magnitude is less than the first rejected member of equation, and has the same sign. If to consider only first two summands in the right part of this formula, then, after certain transformations, the formula (6) in initial notations shall be written as:

$$T=\frac{1}{\lambda_0}\left[1-\frac{1}{2}\cdot\frac{1}{\left(\dfrac{\lambda_0^2}{2\alpha}\right)}\right],\ \text{where}\left(\frac{\lambda_0^2}{2\alpha}>1\right). \qquad (7)$$

According to (7), the mean lifetime of a “linearly ageing” object depends not only on λ_0, but also on the non-dimensional parameter $\lambda_0^2 / 2a$.

To make an obtained result more demonstrative from the physical point of view, let us reduce the expression (7) to the form:

$$T = T_0(1-aT_0^2), \qquad (8)$$

where T_0=$(1/\lambda_0)$ is a mean lifetime of a stationary (“non-ageing”) object with the failure rate λ_0 (see (1) with a=0). The functional connections (8) are qualitatively shown in Fig. 1.

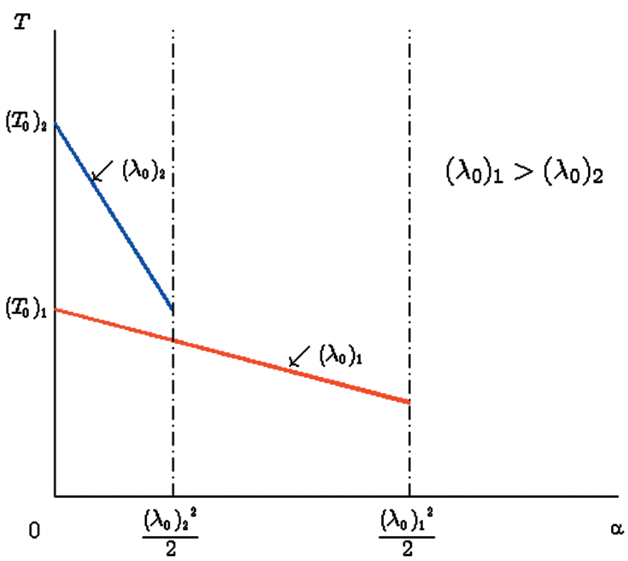

Fig. 1

Fig. 1 shows the reduction of the mean lifetime of a non-stationary objects depending on the ageing factor a.

A possible use of the obtained result shall be analyzed by the following example.

Let us assume that an ageing object, the failure rate of which is expressed by the functional connection (1), is considered as an object with the repair rate μ_0 (which is constant in this case). It is necessary to define how the probabilities of change of operable $p_0(t)$ and non-operable $p_1(t)$ object’s states change with time, and to find its availability rate.

A methodological task seems to have easy solution: by the known rules [3] it is necessary to form a kind of differential Kolmogorov equations in reference to $p_0(t)$ and $p_1(t)$ (with consideration of (1)) and to solve them at the given initial conditions (for instance, with $p_0(0)$=1; $p_1(0)$=0). However, it turned out that if it is relatively simple to write the mentioned equations, it is not possible to find their general solutions. Thus, the assigned task can not be solved in analytical form.

Engineering approach in similar cases involves the search for an approximate solution of the task by means of certain simplified conditions. One of possible ways to realize such approach can be stationarization of the flow of a real non-stationary object, i.e. its replacement with a certain equivalent virtual object with constant rate λ_c=*const*, the value of which is selected based on additional considerations [5, 6]. Then the assigned task is reduced to the form of a stationary one, and its solution does not cause any essential difficulties. But the question of how to define λ_c becomes a question of high priority.

Supposing that λ_c is found based on the condition of equality of the lifetimes of a real (“linearly ageing”) object (T) and its equivalent virtual stationary one $\left(T_c=\dfrac{1}{\lambda_c}\right)$, i.e. based on the correlation T_c=T. If we replace T in the left part of the formula (7) with $1/\lambda_c$ and solve the equation for λ_c, we shall have:

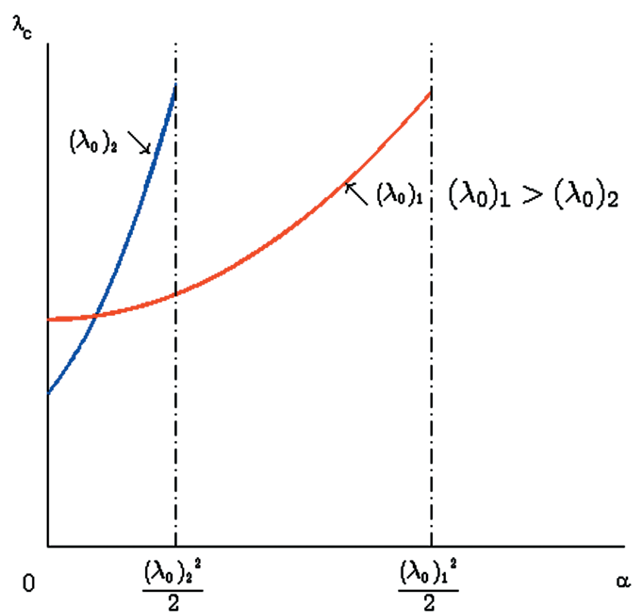

Fig. 2

$$\lambda_c = \lambda_0 \cdot \frac{\lambda_0^2}{\lambda_0^2 - \alpha}; \left(\alpha < \frac{\lambda_0^2}{2} \right), \tag{9}$$

where the application range of the formula (9) is derived from the condition $\frac{\lambda_0^2}{2\alpha} > 1$, accepted at the execution of the real research. The formula (9) shows how λ_c is related to the parameters of $\lambda(t)$ characteristics of a real object. The graphs of this dependency with different values of λ_0 are quantitatively shown in Fig. 2.

After determination of λ_c the $p_0(t)$ value task is reduced to a stationary case; its solution result is known (see, for instance, [7]). If we substitute it with λ_c derived from the formula (9), we shall definitely have:

$$p_0(t) = \frac{\mu_0(\lambda_0^2 - \alpha)}{\lambda_0^3 + \mu_0(\lambda_0^2 - \alpha)} \cdot \left\{ 1 + \frac{\lambda_0^3}{\mu_0(\lambda_0^2 - \alpha)} \cdot \exp\left[-\frac{\lambda_0^3 + \mu_0(\lambda_0^2 - \alpha)}{\lambda_0^2 - \alpha} \cdot t \right] \right\}, \tag{10}$$

$$p_1(t) = \frac{\lambda_0^3}{\lambda_0^3 + \mu_0(\lambda_0^2 - \alpha)} \cdot \left\{ 1 - \exp\left[-\frac{\lambda_0^3 + \mu_0(\lambda_0^2 - \alpha)}{\lambda_0^2 - \alpha} \cdot t \right] \right\}. \tag{11}$$

We use (10) to find the final probability of the operable state $p_0(\infty)$, which does numerically coincide here with an object's availability factor k_Γ:

$$k_\Gamma = \lim_{t \to \infty} p_0(t) = \frac{\mu_0(\lambda_0^2 - \alpha)}{\lambda_0^3 + \mu_0(\lambda_0^2 - \alpha)}.$$

The dependency graphs $k_\Gamma = k_\Gamma(a)$ for different values of λ_0 are quantitatively shown in Fig.3.

As expected from physical views, k_Γ decreases with an ageing coefficient a.

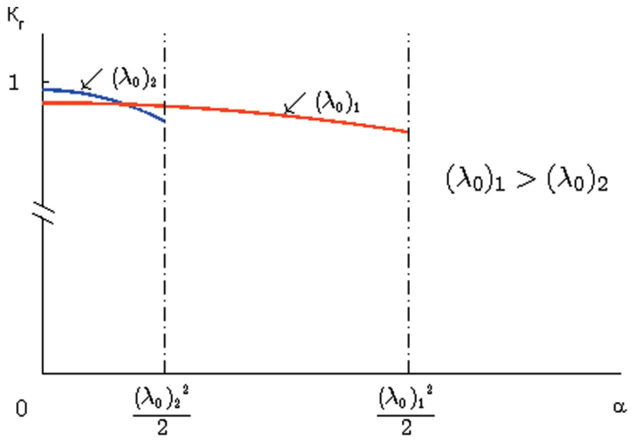

Fig. 3

Therefore, the efficiency of the proposed method for stationarization of the "linearly ageing" object's failure flow is shown by the example of one of possible practical tasks the solution of which is deduced in analytical form.

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