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ESTIMATION OF ECONOMIC EFFICIENCY INDICATORS OF SYSTEMS WITH FUZZY PARAMETERS

The present work offers the method of estimation of economic efficiency indicators of the automated technological facility "protection object – safety system" with recoverable elements considering the uncertainty of a model's parameters. The fuzzy estimation for the average profit of facility operation per time unit is obtained. The example of numerical evaluation of the examined indicator of efficiency with fuzzy parameters is given.

Keywords: *reliability, profit, efficiency, safety system, stochastic variables, a mean lifetime, casual process, mathematical time expectations.*

1. Problem statement

In this paper we will consider the mathematical model allowing estimation of economic efficiency indicators of sequential systems with fuzzy parameters. The sequential systems are the systems, whose state depends on the sequence of faults and recoveries of elements during the period of time from the start of functioning up to the current moment. In the present paper we will consider the system consisting of a protection object and a safety system.

Systems consisting of a protection object and a safety system are applied in cases, where it is required to ensure safe operation of a potentially dangerous object. The safety system is intended to transfer emergency situations under the violation of normal functioning of the protection object in harmless state, i.e. to make adequate action under the fault of the protection object. In such system a vertical hierarchy takes place, as the protection object is under supervision of a safety system. The safety system possesses the interference right to prevent potentially dangerous modifications in the protection object. Interdependence of actions also takes place, as success of operation of the system as a whole and, actually, elements of any level depends on behavior of all system's elements. Thus, it is required to consider the protection object and safety system collectively as a single automated processing facility "protection object – safety system" (APF PO-SS). Mathematical models of reliability of such facility were analyzed in papers [1,2].

Here we will consider a case, when the income from use of APF PO-SS facility and expenses on its service are directly proportionate to time. Correct functioning of protection object brings in some income, while its recovery after the fault requires expenses. The safety system in the course of its operation does not bring income, and preservation of its correct functioning demands spending of some amount, which is also directly proportionate to time of functioning. Based on these suppositions, we will receive an estimation of average profit from operation of the facility during its work till an accident. Considering expenses on recovery of the facility after an accident, we will write the estimation for average profit per a time unit as the result of a facility operation.

During the analysis of the system's process of functioning and calculation of indicators of economic efficiency, there can be an uncertainty of results caused by different reasons, incompleteness of information on research object, various approximations and assumptions during development of mathematical model, approximated methods of calculations and uncertainty of model's parameters. In the present paper we will consider, how the result of analysis is influenced by uncertainty of model's parameters. This variety of uncertainty arises, as parameters of a mathematical model cannot be known precisely owing to insufficiency of data and variability of characteristics.

For uncertainty modeling several approaches different from each other have been developed, for instance, the probability-theoretic approach, fuzzy sets and measures as well as some others, discussion on distinctions and advantages of which could be found in paper [3]. In this paper we will use a combination of probability-theoretic approach and fuzzy measures for construction of a mathematical model considering uncertainty of parameters. Many different fuzzy measures and definitions of fuzzy integral and average of distribution of fuzzy values were offered [4, 5]. In our paper we shall use the measure of probability and average of distribution of fuzzy values on the basis of Choquet integral as it is offered in the works of Liu [6], as such approach goes well with the probability-theoretic approach. For combination within the limits of one model of two types of uncertainty there are also several different approaches offered. In particular, these are fuzzy random variables, random fuzzy variables and hybrid variables [7, 8]. In this paper we will use random fuzzy values, as they make it possible to describe a situation we are interested in the most simple way.

2. Problem solution

Non-failure operating times and times of recovery of the system's elements during development of its mathematical models are described, as a rule, by means of stochastic values. For example, let us consider failure time χ with distribution function $F_\chi(t;\vec{\lambda})$, where $\vec{\lambda}$ is a vector of distribution parameters. Exact values of parameters $\vec{\lambda}$ owing to those or other reasons can be unknown. Thus, uncertainty of model's parameters takes place, which leads to uncertainty of values of required indicators of efficiency. For quantitative description of the specified uncertainty we will take the advantage of mathematical apparatus of random fuzzy values [7, 8]. The essence of applied approach consists in the fact that random variables receive a measure of probability [6]. Random fuzzy failure times and recovery times will be defined using the scheme offered in [7]. In order to set random fuzzy value χ, we will specify a set of probability distributions $\left\{F_\chi\left(t;\vec{\lambda}(\theta)\right), \theta\in\Theta\right\}$ in probability space (Ω,A,P), where $\vec{\lambda}$ is the fuzzy vector defined in space of probability (Θ,Π,Cr), to which the membership function $\mu_{\vec{\lambda}}(\vec{x})$ corresponds. For example, χ~$EXP(\lambda)$, if

$$F_\chi\left(t;\lambda(\theta)\right)=\begin{cases}1-e^{-\lambda(\theta)t}, \text{ if } t\geq 0,\\ 0, \text{ otherwise,}\end{cases}$$

where λ is a fuzzy value with membership function $\mu_\lambda(x)$.

In [7] it is shown that if there is a set random fuzzy value χ, then $P(\chi\in A)$, where $A\subseteq R$ and $M\chi$ are fuzzy values. Thus, parameters of the considered mathematical model are parameters of distributions of failure times and recovery times, and their uncertainty is described by means of the corresponding membership function. Having set the random fuzzy failure times and recovery times, we can receive membership functions also for economic efficiency indicators. For comparison of the received results with results received by classical methods, it is possible to take advantage of defuzzification procedure, as a result of application of which a well-defined characterizing value is put in correspondence to a fuzzy value. For this purpose, in this paper we will use the average of distribution of random fuzzy values, as it is described in [6]. Such approach is combined in the best way with use of random fuzzy values for description of failure times and times of recovery.

It is well known that for description of process of functioning of restored elements it is possible to use alternating processes of recovery, as well as accumulation processes [9, 10, 11]. In papers [1, 2] the process of APF PO-SS functioning is described by means of superposition of alternating processes of recovery.

As in this paper we go from random variables to random fuzzy variables, it is necessary to consider the random fuzzy process of recovery and random fuzzy process of accumulation, which were offered in papers [4, 6].

Let us describe now in more detail the process of functioning of the considered facility within the framework of the offered model. We will designate the operating time to the first failure of protection object χ_1, failure time of a protection object after its first recovery we will designate as χ_2, after the second recovery as χ_3, etc. As we consider regenerating process, we will assume that all χ_i are equally distributed and independent. Time of recovery after the first fault of a protection object we will designate γ_1, time of recovery after its second fault γ_2, etc. All random variables γ_i we consider also equally distributed and independent. We will consider two types of faults of safety system: latent faults and false faults. Latent faults of a safety system are such faults, which cannot be discovered without carrying out special actions for monitoring of working capacity of safety system. False faults of a safety system are such faults, which lead to spontaneous protective action at fault-free protection object. We will designate the operating time to the first latent fault of safety system as ξ_1, we will designate operating time to latent fault of safety system after its first recovery as ξ_2, after the second recovery as ξ_3, etc. All ξ_i are equally distributed and independent. Time of recovery of safety system after detection of its first latent fault we will designate as η_1, time of recovery after detection of its second latent fault as η_2, etc. All η_i are also equally distributed and independent. We will designate operating time to the first false fault of safety system as φ_1, operating time to false fault of safety system after the first recovery we will designate as φ_2, after the second recovery as φ_3, etc. All φ_i are independent and equally distributed. Time of recovery after the first false fault of a safety system we will designate as ψ_1, after the second false fault as ψ_2, etc. All ψ_i are also independent and equally distributed. As the safety system functions in a mode of expectation of fault of protection object, it is impossible to discover its latent faults at the moment of their origination. Therefore, for detection of latent faults the procedure of periodic control of the safety system functionality is introduced. We will specify the period functionality control as T, and its duration as δ. For the time of periodic control, the safety system ceases to fulfill its functions. Thus, the facility accident happens in the case, when a fault of the protection object occurs during a disabled condition of the safety system. The moments of regeneration of the facility functioning are the moments of termination of recovery of the protection object, during these moments the facility as though forgets its past and returns to the original state. The number of regeneration cycle of process of functioning of the facility, on which there was an accident, we will designate as v. According to the reasons given above, values χ_i, γ_i, ξ_i, η_i, φ_i, ψ_i and v are random fuzzy.

As we already mentioned above, the income from operation and expenses on service of the facility are directly proportionate to time. We will introduce the following designations: C_0 is the income per time unit of the protection object functioning, C_1 are expenses per time unit on recovery of the protection object after its fault, C_2 are expenses per time unit on recovery after false fault of a safety system, C_3 are expenses per time unit on functioning of a safety system, C_4 are expenses per time unit on monitoring of safety system, C_5 are expenses per time unit on recovery of as safety system after the latent fault.

It is shown that random fuzzy operating time ω of facility to accident is defined as follows [2]:

$$\omega = \sum_{i=1}^{\nu-1} \tau_i + \tau'_\nu,$$

where τ_i is random fuzzy duration of regeneration cycle of process of functioning of facility, on which there was no accident, and τ_i' is random fuzzy duration of regeneration cycle of process of functioning of facility, on which there was an accident. Then also for random fuzzy profit ρ during the operation of a facility before the accident it is fair to consider

$$\rho = \sum_{i=1}^{\nu-1} \sigma_i + \sigma'_\nu,$$

where σ_i is a random fuzzy profit for one cycle of regeneration of process of the functioning of a facility, on which there was no accident, and σ_i' is a random fuzzy profit for one cycle of regeneration of process of the functioning of a facility, on which there was an accident.

Then we will take advantage of the fact that for every fixed $\theta \in \Theta$ values $M\omega(\theta)$, $M\tau_i(\theta)$, $M\tau_i'(\theta)$, $M\rho(\theta)$, $M\sigma_i(\theta)$, $M\sigma_i'(\theta)$ and $Mv(\theta)$ are well-defined values [6]. For every fixed $\theta \in \Theta$ it is fair to consider

$$M\omega(\theta) = M\left(\sum_{i=1}^{\nu(\theta)-1} \tau_i(\theta) + \tau'_{\nu(\theta)}(\theta)\right) \text{ and}$$

$$M\sigma(\theta) = M\left(\sum_{i=1}^{\nu(\theta)-1} \sigma_i(\theta) + \sigma'_{\nu(\theta)}(\theta)\right).$$

As, the considered random variables for every fixed $\theta \in \Theta$ are independent, using the composite probability formula, we can write:

$$P\left(\sum_{i=1}^{\nu(\theta)-1} \sigma_i(\theta) + \sigma'_{\nu(\theta)}(\theta) \geq r\right) =$$

$$= \sum_{k=1}^{\infty} P(\nu(\theta) = k) P\left(\sum_{i=1}^{k-1} \sigma_i(\theta) + \sigma'_k(\theta) \geq r\right) \text{ and}$$

$$M\rho(\theta) = \int_0^{\infty} P\left(\sum_{i=1}^{\nu(\theta)-1} \sigma_i(\theta) + \sigma'_{\nu(\theta)}(\theta) \geq r\right) dr =$$

$$= \sum_{k=1}^{\infty} P(\nu(\theta) = k) M\left(\sum_{i=1}^{k-1} \sigma_i(\theta) + \sigma'_k(\theta)\right).$$

Taking into account that the considered values are independent and equally distributed, we receive

$$M\rho(\theta) = \sum_{k=1}^{\infty} P(\nu(\theta) = k) M(\sigma'(\theta) + (k-1)\sigma(\theta)) =$$

$$= M(\sigma'(\theta) + (\nu(\theta) - 1)\sigma(\theta)).$$

As the functioning of a technological facility's safety system is a regenerating process, the probability that there will be a facility fault in the k-th cycle of regeneration can be written as

$$P(v(\theta) = k) = r(\theta)(1 - r(\theta))^{k-1},$$

where $r(\theta)$ is the probability of a facility's failure in intervals of recovery of a safety system. Then the random fuzzy operating time $\omega(\theta)$ and random fuzzy profit $\rho(\theta)$ for the operating time of a facility before an accident provided that the considered values are independent and equally distributed, we can rewrite for every fixed $\theta \in \Theta$:

$$M\omega(\theta) = M\tau'(\theta) + \frac{1 - r(\theta)}{r(\theta)} M\tau(\theta) \text{ and}$$

$$M\rho(\theta) = M\sigma'(\theta) + \frac{1 - r(\theta)}{r(\theta)} M\sigma(\theta),$$

where $r(\theta)$ is the probability of an accident at the regeneration cycle of the functioning of a facility.

Now we will consider in more detail the duration of regeneration cycles and profits corresponding to them. The average duration of the regeneration cycle of the functioning process of a facility, on which there was no accident, for every $\theta \in \Theta$, we will define as follows:

$$M\tau(\theta)=M\left(\min(\chi(\theta),\phi(\theta))+\gamma(\theta)J_{\chi(\theta)<\phi(\theta)}+\psi(\theta)J_{\phi(\theta)\le\chi(\theta)}\right),$$

where J_A is indicator of event A. The average duration of the regeneration cycle of the functioning process of a facility, on which there was an accident, for every fixed $\theta\in\Theta$ is equal to $M(\tau'(\theta)) = M(\chi(\theta))$. The fuzzy average of the distribution of profit for one cycle of regeneration of a facility's functioning, on which there was no accident, for every fixed $\theta\in\Theta$ looks like

$$M(\sigma(\theta))=M\begin{pmatrix}(C_0-C_{SS}(\theta))\min(\chi(\theta),\phi(\theta))-\\ -C_1\gamma(\theta)J_{\chi(\theta)<\phi(\theta)}-C_2\psi(\theta)J_{\phi(\theta)\le\chi(\theta)}\end{pmatrix},$$

where $C_{SS}(\theta)$ are expenses per time unit of the safety system operation.

The fuzzy average of distribution of profit for one cycle of regeneration of process of functioning of facility, on which there was an accident, for every fixed $\theta\in\Theta$ is written as

$$M(\sigma'(\theta)) = M((C_0-C_{ss}(\theta))\chi(\theta)).$$

Then, fulfilling simple transformations, we will write for every $\theta\in\Theta$

$$M(\sigma(\theta))=(C_0-C_{SS}(\theta))\int_0^\infty\left(1-F_\chi(t;\vec\lambda_\chi(\theta))\right)\left(1-F_\phi(t;\vec\lambda_\phi(\theta))\right)dt-$$
$$-C_1\int_0^\infty\left(1-F_\gamma(t;\vec\lambda_\gamma(\theta))\right)dt\left(1-\int_0^\infty F_\phi(t;\vec\lambda_\phi(\theta))dF_\chi(t;\vec\lambda_\chi(\theta))\right)-$$
$$-C_2\int_0^\infty\left(1-F_\psi(t;\vec\lambda_\psi(\theta))\right)dt\int_0^\infty F_\phi(t;\vec\lambda_\phi(\theta))dF_\chi(t;\vec\lambda_\chi(\theta)),$$

$$M(\sigma'(\theta))=(C_0-C_{SS}(\theta))\int_0^\infty\left(1-F_\chi(t;\vec\lambda_\chi(\theta))\right)dt,$$

$$M(\tau(\theta))=\int_0^\infty\left(1-F_\chi(t;\vec\lambda_\chi(\theta))\right)\left(1-F_\phi(t;\vec\lambda_\phi(\theta))\right)dt+$$
$$+\int_0^\infty\left(1-F_\gamma(t;\vec\lambda_\gamma(\theta))\right)dt\left(1-\int_0^\infty F_\phi(t;\vec\lambda_\phi(\theta))dF_\chi(t;\vec\lambda_\chi(\theta))\right)+$$
$$+\int_0^\infty\left(1-F_\psi(t;\vec\lambda_\psi(\theta))\right)dt\int_0^\infty F_\phi(t;\vec\lambda_\phi(\theta))dF_\chi(t;\vec\lambda_\chi(\theta)),$$

$$M(\tau'(\theta))=\int_0^\infty\left(1-F_\chi(t;\vec\lambda_\chi(\theta))\right)dt.$$

In order to calculate averages of distribution $M(\omega(\theta))$ and $M(\rho(\theta))$, it is necessary for us to receive a correlation for probability $r(\theta)$at every fixed $\theta\in\Theta$. Designating probability of malfunctioning of safety system $q(\theta)$, we receive

$$r(\theta)=P(\chi(\theta)<\phi(\theta))q(\theta)=$$
$$=\left(1-\int_0^\infty F_\phi(t;\vec\lambda_\phi(\theta))dF_\chi(t;\vec\lambda_\chi(\theta))\right)q(\theta).$$

Hence, the facility accident happens, when a protection object's failure happened before a safety system's false failure of, and the safety system did not initiate protective action owing to its latent fault. We will write that the considered events are statistically independent. We will calculate now a correlation for probability $q(\theta)$. We will designate $Q^+(\theta)$ − a set of those instants, during which duration the safety system is capable to parry the fault of protection object, and $Q^-(\theta)$ − a set of instants, during which duration the safety system is not capable to parry fault of protection object. Probability $q(\theta)$is calculated as follows:

$$q(\theta)=\int_0^\infty P(t\in Q^-(\theta))dF_\chi(t;\vec\lambda_\chi(\theta)).$$

Hence, it is necessary to calculate probability $P(t\in Q^-(\theta))$ for every fixed $\theta\in\Theta$. We will write

$$P(t\in Q^-(\theta))=1-P(t\in Q^+(\theta))=1-P^+(t;\theta).$$

According to the composite probability formula we receive

$$P^+(t;\theta)=\int_0^\infty\int_0^\infty P(t\in Q^+(\theta)\,|\,\xi(\theta)=x,\eta(\theta)=y)\cdot$$
$$\cdot dF_\eta(y;\vec\lambda_\eta(\theta))dF_\xi(x;\vec\lambda_\xi(\theta)).$$

The process of functioning of a periodically controllable safety system is regenerating with the period of regeneration $\tau_{SS}(\xi,\eta)=\left(\left[\frac{\xi}{T+\delta}\right]+1\right)(T+\delta)+\eta$, where $[x]$ is the whole part x. Then we can write:

$$P^+(t;\theta)=\iint_{\tau_{SS}(x,y)\le t}\begin{pmatrix}P(t\in Q^+(\theta)\,|\,\xi(\theta)=x,\eta(\theta)=y)\\ dF_\eta(y;\vec\lambda_\eta(\theta))dF_\xi(x;\vec\lambda_\xi(\theta))\end{pmatrix}+$$
$$+\iint_{\tau_{SS}(x,y)>t}\begin{pmatrix}P(t\in Q^+(\theta)\,|\,\xi(\theta)=x,\eta(\theta)=y)\\ dF_\eta(y;\vec\lambda_\eta(\theta))dF_\xi(x;\vec\lambda_\xi(\theta))\end{pmatrix}=I_1+I_2.$$

Let us calculate at first I_2:

$$I_2=\iint_{\tau_{SS}(x,y)>t}\left(\sum_{m=0}^{\left[\frac{x}{T+\delta}\right]-1}J_{t\in[m(T+\delta),m(T+\delta)+T)}+J_{t\in\left[\left[\frac{x}{T+\delta}\right](T+\delta),x\right)}\right)\cdot$$
$$\cdot dF_\eta(y;\vec\lambda_\eta(\theta))dF_\xi(x;\vec\lambda_\xi(\theta)),$$

where $J_{t\in A}$ is the indicator of event $t\in A$. If we omit some simple, but cumbersome transformations, we will write at once result:

$$I_2 = \left(1 - F_\xi\left(t;\vec{\lambda}_\xi(\theta)\right)\right) - \sum_{m=1}^{\infty} \begin{matrix}\left(1 - F_\xi\left(m(T+\delta);\vec{\lambda}_\xi(\theta)\right)\right)\cdot \\ \cdot\left(J_{(m-1)(T+\delta)+T\le t} - J_{m(T+\delta)\le t}\right)\end{matrix} = \\ = F_\zeta\left(t;\vec{\lambda}_\xi(\theta)\right) - F_\xi\left(t;\vec{\lambda}_\xi(\theta)\right),$$

where

$$F_\zeta\left(t;\vec{\lambda}_\xi(\theta)\right) = 1 - \sum_{m=1}^{\infty} \begin{matrix}\left(1 - F_\xi\left(m(T+\delta);\vec{\lambda}_\xi(\theta)\right)\right) \\ \left(J_{(m-1)(T+\delta)+T\le t} - J_{m(T+\delta)\le t}\right)\end{matrix}$$

is the cumulative distribution function of some auxiliary random variable ζ. We will consider now item I_1:

$$I_1 = \iint_{\tau_{СБ}(x,y)\le t} P^+\left(t - \tau_{СБ}(x,y)\right) dF_\eta\left(y;\vec{\lambda}_\eta(\theta)\right) dF_\xi\left(x;\vec{\lambda}_\xi(\theta)\right) = \\ = \int_0^t P^+(t-z)\, dF_{\tau_{СБ}}\left(z;\vec{\lambda}_\eta(\theta),\vec{\lambda}_\xi(\theta)\right).$$

Thus, summarizing I_1 and I_2, at every fixed $\theta\in\Theta$ we receive an integral equation for $P(t\in Q^+(\theta))$:

$$P^+(t;\theta) = f(t;\theta) + \int_0^t P^+(t-z)\, dF_{\tau_{ss}}\left(z;\vec{\lambda}_\eta(\theta),\vec{\lambda}_\xi(\theta)\right), \quad (1)$$

where

$$F_{\tau_{ss}}\left(z;\vec{\lambda}_\eta(\theta),\vec{\lambda}_\xi(\theta)\right) = P\left(\left(\left[\frac{\xi(\theta)}{T+\delta}\right]+1\right)(T+\delta) + \eta(\theta) \le z\right)$$

is the cumulative distribution function of random variable τ_{ss},

$$f(t;\theta) = F_\zeta\left(t,\vec{\lambda}_\xi(\theta)\right) - F_\xi\left(t,\vec{\lambda}_\xi(\theta)\right).$$

Using Laplace-Stieltjes transform and tauberian theorems, it is easy to receive an asymptotic solution of equation (1):

$$\lim_{t\to\infty} P^+(t;\theta) = \frac{M\xi(\theta) - M\zeta(\theta)}{M\tau_{СБ}(\theta)} = 1 - q(\theta).$$

If we omit simple calculations, we will reduce expressions for averages of distribution $M\zeta(\theta)$, $M\xi(\theta)$ and $M\tau_{SS}(\theta)$:

$$M\zeta(\theta) = \delta M\left[\frac{\xi(\theta)}{T+\delta}\right] = \\ = \delta\sum_{k=1}^{\infty} k\left(F_\xi\left((k+1)(T+\delta);\vec{\lambda}_\xi(\theta)\right) - F_\xi\left(k(T+\delta);\vec{\lambda}_\xi(\theta)\right)\right),$$

$$M\xi(\theta) = \int_0^\infty \left(1 - F_\xi\left(t;\vec{\lambda}_\xi(\theta)\right)\right) dt,$$

$$M\tau_{SS}(\theta) = \int_0^\infty \left(1 - F_\eta\left(t;\vec{\lambda}_\eta(\theta)\right)\right) dt + (T+\delta)\cdot \\ \cdot\left(1 + \sum_{k=1}^{\infty} k\left(F_\xi\left((k+1)(T+\delta);\vec{\lambda}_\xi(\theta)\right) - F_\xi\left(k(T+\delta);\vec{\lambda}_\xi(\theta)\right)\right)\right).$$

Now let us calculate the expenses on operation of a safety system $C_{SS}(\theta)$ for every fixed $\theta\in\Theta$. We will consider the process of functioning of a safety system. The process of functioning of a safety system is an alternating process of recovery with a period of regeneration

$$\tau_{SS}(\xi,\eta) = (T+\delta)\left(\left[\frac{\xi}{T+\delta}\right]+1\right) + \eta.$$

Using the formula of composite probability and designating expenses on operation of safety system $C_{SS}(t;\theta)$ to point of time t we receive:

$$C_{SS}(t;\theta) = \int_0^\infty\int_0^\infty C_{SS}(t;\theta)\Big|_{\xi(\theta)=x,\eta(\theta)=y} dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right).$$

When deducting $C_{SS}(t;\theta)$, it is necessary to consider two cases: first, when $\tau_{SS}(x, y)\le t$, i.e. the first cycle of regeneration ended before point of time t and, second, when $\tau_{SS}(x, y)>t$ and therefore, the first cycle of regeneration of process of safety system functioning did not end before point of time t. Then we will write

$$C_{SS}(t;\theta) = \iint_{\tau_{SS}(x,y)\le t} C_{SS}(t;\theta)\Big|_{\xi(\theta)=x,\eta(\theta)=y} dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right) + \\ + \iint_{\tau_{SS}(x,y)>t} C_{SS}(t;\theta)\Big|_{\xi(\theta)=x,\eta(\theta)=y} dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right) = S_1 + S_2.$$

At first let us calculate S_1:

$$S_1 = \iint_{\tau_{SS}(x,y)\le t} \begin{pmatrix} C_3\left(\left[\frac{x}{T+\delta}\right]+1\right)T + C_4\left(\left[\frac{x}{T+\delta}\right]+1\right)\delta + \\ +C_5 y + C_{SS}(t-\tau_{SS}(x,y);\theta)\end{pmatrix}\cdot \\ \cdot dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right) = \\ = \iint_{\tau_{SS}(x,y)\le t} \begin{pmatrix} C_3\left(\left[\frac{x}{T+\delta}\right]+1\right)T + \\ +C_4\left(\left[\frac{x}{T+\delta}\right]+1\right)\delta + C_5 y\end{pmatrix} dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right) + \\ + \int_0^t C_{SS}(t-z;\theta)\, dF_{\tau_{SS}}\left(z;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right)$$

Then let us calculate S_2:

$$S_2 = \iint\limits_{\substack{\tau_{SS}(x,y)>t \\ \left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)>t}} \left(C_3\left[\frac{t}{T+\delta}\right]T + C_4\left[\frac{t}{T+\delta}\right]\delta + \right.$$

$$+C_3\left(t-\left[\frac{t}{T+\delta}\right](T+\delta)\right)J_{t-\left[\frac{t}{T+\delta}\right](T+\delta)\le T} +$$

$$\left. +\left(C_3T + C_4\left(t-\left[\frac{t}{T+\delta}\right](T+\delta)-T\right)^+\right)J_{t-\left[\frac{t}{T+\delta}\right](T+\delta)>T}\right)\cdot$$

$$\cdot dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right)+$$

$$+\iint\limits_{\substack{\tau_{SS}(x,y)>t \\ \left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)\le t}} \begin{pmatrix} C_3\left(\left[\frac{x}{T+\delta}\right]+1\right)T + C_4\left(\left[\frac{x}{T+\delta}\right]+1\right)\delta + \\ +C_5\left(t-\left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)\right)^+ \end{pmatrix}\cdot$$

$$\cdot dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right),$$

where $x^+ = \max(x,0)$. Summarizing S_2 and S_1, we receive the equation for average summarized expenses on operation of safety system $C_{SS}(t;\theta)$ by point of time t

$$C_{SS}(t;\theta) = g(t;\theta) + \int_0^t C_{SS}(t-z;\theta)\,dF_{\tau_{SS}}\left(z;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right), \quad (2)$$

Where

$$g(t;\theta) = C_3\left(\int\limits_{\left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)\le t} \left(\left[\frac{x}{T+\delta}\right]+1\right)T\,dF_\xi\left(x;\vec{\lambda}_\xi(\theta)\right)+ \right.$$

$$+\left(\left[\frac{t}{T+\delta}\right]T + \left(t-\left[\frac{t}{T+\delta}\right](T+\delta)\right)J_{t-\left[\frac{t}{T+\delta}\right](T+\delta)\le T} + \right.$$

$$\left.\left. +TJ_{t-\left[\frac{t}{T+\delta}\right](T+\delta)>T}\right)\int\limits_{\left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)>t} dF_\xi\left(x;\vec{\lambda}_\xi(\theta)\right)\right)+$$

$$+C_4\left(\int\limits_{\left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)\le t}\left(\left[\frac{x}{T+\delta}\right]+1\right)\delta\,dF_\xi\left(x;\vec{\lambda}_\xi(\theta)\right)+\left(\left[\frac{t}{T+\delta}\right]\delta + \right.\right.$$

$$\left. +\left(t-\left[\frac{t}{T+\delta}\right](T+\delta)-T\right)^+ J_{t-\left[\frac{t}{T+\delta}\right](T+\delta)>T}\right)\cdot$$

$$\left.\cdot\int\limits_{\left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)>t} dF_\xi\left(x;\vec{\lambda}_\xi(\theta)\right)\right)+C_5\left(\iint\limits_{\tau_{CB}(x,y)\le t} y\,dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right)+\right.$$

$$\left.+\iint\limits_{\substack{\tau_{CB}(x,y)>t \\ \left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)\le t}}\left(t-\left(\left[\frac{x}{T+\delta}\right]+1\right)(T+\delta)\right)^+ dF_{\xi,\eta}\left(x,y;\vec{\lambda}_\xi(\theta),\vec{\lambda}_\eta(\theta)\right)\right).$$

The solution of equation (2) will be found with the application of Laplace-Stieltjes transform and corresponding limiting theorems. As a result we receive an asymptotic solution of equation (2) as:

$$C_{SS}(\theta) \approx \lim_{t\to\infty}\frac{C_{SS}(t;\theta)}{t} =$$

$$= \frac{C_3TM\alpha(\theta) + C_4\delta M\alpha(\theta) + C_5M\eta(\theta)}{M\tau_{SS}(\theta)},$$

where

$$M\alpha(\theta) = M\left(\left[\frac{\xi(\theta)}{T+\delta}\right]+1\right) = 1 +$$

$$+\sum_{k=1}^{\infty} k\left(F_\xi\left((k+1)(T+\delta);\vec{\lambda}_\xi(\theta)\right) - F_\xi\left(k(T+\delta);\vec{\lambda}_\xi(\theta)\right)\right).$$

Thus, having received all necessary relations, we can calculate now averages of distribution $M\omega(\theta)$ and $M\rho(\theta)$ at every fixed $\theta\in\Theta$, and it means that according to definition of function from fuzzy values [6], we set fuzzy average of distribution of operating time to the first accident and fuzzy average of distribution of profit for operating time of facility before accident as functions from model's fuzzy parameters. We will write now expression for average profit per time unitfrom facility operation. Applying the well-known formula for process of accumulation [11], we will write for every fixed $\theta\in\Theta$:

$$C_{ATF}(\theta) = \frac{M\rho(\theta) - C_\beta\int_0^\infty\left(1-F_\beta\left(t;\vec{\lambda}_\beta(\beta)\right)\right)dt}{M\omega(\theta) + \int_0^\infty\left(1-F_\beta\left(t;\vec{\lambda}_\beta(\beta)\right)\right)dt},$$

where β is random fuzzy time of recovery of a facility after accident, C_β are expenses per time unit on recovery of facility after accident

In order to write correlations for membership functions of values $M\omega$, $M\rho$ and C_{ATF} we will take advantage of enlargement principle [6]. So function of membership of value $M\omega$ is:

$$\mu_{M\omega}(y) = \sup_{y=f_1(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6)} \min\left(\mu_{\vec{\lambda}_\chi}(\vec{x}_1),\mu_{\vec{\lambda}_\gamma}(\vec{x}_2),\mu_{\vec{\lambda}_\xi}(\vec{x}_3),\right.$$

$$\left.\mu_{\vec{\lambda}_\eta}(\vec{x}_4),\mu_{\vec{\lambda}_\phi}(\vec{x}_5),\mu_{\vec{\lambda}_\psi}(\vec{x}_6)\right),$$

where

$$f_1(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6) =$$

$$= M\tau'(\vec{x}_1) + \frac{1-r(\vec{x}_1,\vec{x}_3,\vec{x}_4,\vec{x}_5)}{r(\vec{x}_1,\vec{x}_3,\vec{x}_4,\vec{x}_5)}M\tau(\vec{x}_1,\vec{x}_2,\vec{x}_5,\vec{x}_6),$$

$$M\tau(\vec{x}_1,\vec{x}_2,\vec{x}_5,\vec{x}_6) = \int_0^\infty\left(1-F_\chi(t;\vec{x}_1)\right)\left(1-F_\phi(t;\vec{x}_5)\right)dt +$$

$$+\int_0^\infty\left(1-F_\gamma(t;\vec{x}_2)\right)dt\left(1-\int_0^\infty F_\phi(t;\vec{x}_5)\,dF_\chi(t;\vec{x}_1)\right)+$$

$$+\int_0^\infty\left(1-F_\psi(t;\vec{x}_6)\right)dt\int_0^\infty F_\phi(t;\vec{x}_5)\,dF_\chi(t;\vec{x}_1),$$

$$M\tau'(\vec{x}_1)=\int_0^\infty\left(1-F_\chi(t;\vec{x}_1)\right)dt,$$

$$r(\vec{x}_1,\vec{x}_3,\vec{x}_4,\vec{x}_5)=\left(1-\int_0^\infty F_\phi(t;\vec{x}_5)dF_\chi(t;\vec{x}_1)\right)q(\vec{x}_3,\vec{x}_4),$$

$$q(\vec{x}_3,\vec{x}_4)\approx 1-\frac{M\xi(\vec{x}_3)-M\zeta(\vec{x}_3)}{M\tau_{\tilde{N}\acute{A}}(\vec{x}_3,\vec{x}_4)},$$

$$M\zeta(\vec{x}_3)=\delta\sum_{k=1}^\infty k\left(F_\xi\left((k+1)(T+\delta);\vec{x}_3\right)-F_\xi\left(k(T+\delta);\vec{x}_3\right)\right),$$

$$M\xi(\vec{x}_3)=\int_0^\infty\left(1-F_\xi(t;\vec{x}_3)\right)dt,$$

$$M\tau_{\text{СБ}}(\vec{x}_3,\vec{x}_4)=\int_0^\infty\left(1-F_\eta(t;\vec{x}_4)\right)dt+(T+\delta)\cdot$$
$$\cdot\left(1+\sum_{k=1}^\infty k\left(F_\xi\left((k+1)(T+\delta);\vec{x}_3\right)-F_\xi\left(k(T+\delta);\vec{x}_3\right)\right)\right).$$

Let us write similarly membership function $M\rho$

$$\mu_{M\rho}(y)=\sup_{y=f_2(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6)}\min\left(\mu_{\tilde{\lambda}_\chi}(\vec{x}_1),\mu_{\tilde{\lambda}_\gamma}(\vec{x}_2),\mu_{\tilde{\lambda}_\xi}(\vec{x}_3),\right.$$
$$\left.\mu_{\tilde{\lambda}_\eta}(\vec{x}_4),\mu_{\tilde{\lambda}_\phi}(\vec{x}_5),\mu_{\tilde{\lambda}_\psi}(\vec{x}_6)\right),$$

where

$$f_2(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6)=M\sigma'(\vec{x}_1)+$$
$$+\frac{1-r(\vec{x}_1,\vec{x}_3,\vec{x}_4,\vec{x}_5)}{r(\vec{x}_1,\vec{x}_3,\vec{x}_4,\vec{x}_5)}M\sigma(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6),$$

$$M\sigma(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6)=\left(C_0-C_{\text{СБ}}(\vec{x}_3,\vec{x}_4)\right)\cdot$$
$$\cdot\int_0^\infty\left(1-F_\chi(t;\vec{x}_1)\right)\left(1-F_\phi(t;\vec{x}_5)\right)dt-C_1\cdot$$
$$\cdot\int_0^\infty\left(1-F_\gamma(t;\vec{x}_2)\right)dt\left(1-\int_0^\infty F_\phi(t;\vec{x}_5)dF_\chi(t;\vec{x}_1)\right)-$$
$$-C_2\int_0^\infty\left(1-F_\psi(t;\vec{x}_6)\right)dt\int_0^\infty F_\phi(t;\vec{x}_5)dF_\chi(t;\vec{x}_1),$$

$$M\sigma'(\vec{x}_1,\vec{x}_3,\vec{x}_4)=\left(C_0-C_{\text{SS}}(\vec{x}_3,\vec{x}_4)\right)\int_0^\infty\left(1-F_\chi(t;\vec{x}_1)\right)dt,$$

$$C_{\text{SS}}(\vec{x}_3,\vec{x}_4)\approx$$
$$\approx\frac{C_3TM\alpha(\vec{x}_3)+C_4\delta M\alpha(\vec{x}_3)+C_5\int_0^\infty\left(1-F_\eta(t;\vec{x}_4)\right)dt}{M\tau_{\text{SS}}(\vec{x}_3,\vec{x}_4)},$$

$$M\alpha(\vec{x}_3)=1+$$
$$+\sum_{k=1}^\infty k\left(F_\xi\left((k+1)(T+\delta);\vec{x}_3\right)-F_\xi\left(k(T+\delta);\vec{x}_3\right)\right).$$

And for C_{ATF} we will write:

$$\mu_{C_{\text{ATF}}}(y)=\sup_{y=f_3(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6,\vec{x}_7)}\min\left(\mu_{\tilde{\lambda}_\chi}(\vec{x}_1),\mu_{\tilde{\lambda}_\gamma}(\vec{x}_2),\right.$$
$$\left.\mu_{\tilde{\lambda}_\xi}(\vec{x}_3),\mu_{\tilde{\lambda}_\eta}(\vec{x}_4),\mu_{\tilde{\lambda}_\phi}(\vec{x}_5),\mu_{\tilde{\lambda}_\psi}(\vec{x}_6),\mu_{\tilde{\lambda}_\alpha}(\vec{x}_7)\right),$$

where

$$f_3(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6,\vec{x}_7)=$$
$$=\frac{f_2(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6)-C_\beta\int_0^\infty\left(1-F_\beta(t;\vec{x}_7)\right)dt}{f_1(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4,\vec{x}_5,\vec{x}_6)+\int_0^\infty\left(1-F_\beta(t;\vec{x}_7)\right)dt}.$$

Thus, we have managed to write correlations for target membership function via membership functions of model's parameters.

Let us consider now procedure of defuzzification. As we mentioned earlier, we will take advantage for this purpose of concept of average of distribution of random fuzzy values [7]:

$$E[\rho]=\int_0^\infty Cr\{\theta\in\Theta\mid M\rho(\theta)\geq r\}dr-$$
$$-\int_{-\infty}^0 Cr\{\theta\in\Theta\mid M\rho(\theta)\leq r\}dr.$$

To find the corresponding measure of probability, it is necessary to use the following correlation between measure of probability and membership function [6]:

$$Cr\{M\rho\in B\}=\frac{1}{2}\left(\sup_{y\in B}\mu_{M\rho}(y)+1-\sup_{y\in R\setminus B}\mu_{M\rho}(y)\right).$$

Similarly there are calculated $E[\omega]$ and $E[C_{ATF}]$.

It is easy to notice that the basic complexity in this case is calculation of membership functions. Various methods for solution of this problem were offered, for example, interval arithmetics. However, the best combination of universality and simplicity of realization, in our opinion, is ensured by the generalized method of transformation which was offered by Hanss [12]. This method makes it possible to calculate membership functions for functions from fuzzy values. Thus, it is possible to consider both monotone and nonmonotone functions from fuzzy values. The essence of this method consists in decomposition of membership functions of arguments on б-level sets, calculation of values of function on the received range of points and subsequent reconstruction of the required membership function. Advantages of the specified method is also the fact that on its basis it is possible to estimate contribution of each of fuzzy arguments into total uncertainty of result [12].

It is necessary also to pay attention to a problem of selection of membership functions of fuzzy values. This problem is, generally speaking, quite complicated. Various expert methods, the review of which can be found in [6], are developed for construction of membership functions. However, this approach operates not by objective data received during investigation of system, but by the judgments of experts on the investigated system. In this situation, it seems more feasible to use the method of construction of membership functions offered by Buckley [13]. Its essence consists in the fact that membership function of required distribution parameter is defined by sets of б-level. As a set of б-level, there is taken an interval estimation of required distribution parameter with level of trust($1-\alpha$). The received estimation is more obvious and contains more information on the estimated parameter rather than the point estimation or unique confidential interval.

Finalizing, we will consider numerical example. Let $\chi \sim EXP(\lambda_\chi)$, $\gamma \sim EXP(\lambda_\gamma)$, $\xi \sim EXP(\lambda_\xi)$, $\eta \sim EXP(\lambda_\eta)$, $\varphi \sim EXP(\lambda_\varphi)$, $\psi \sim EXP(\lambda_\psi)$, $\beta \sim EXP(\lambda_\beta)$. And $\mu_{\lambda_\chi}(x) = \Delta(1\times10^{-6}h^{-1}; 1{,}5\times10^{-6}h^{-1}; 2\times10^{-6}h^{-1})$, $\mu_{\lambda_\xi}(x) = \Delta(1\times10^{-4}h^{-1}; 1{,}5\times10^{-4}h^{-1}; 2\times10^{-4}h^{-1})$, $\mu_{\lambda_\xi}(x) = \Delta(1\times10^{-4}h^{-1}; 1{,}5\times10^{-4}h^{-1}; 2\times10^{-4}h^{-1})$, $\mu_{\lambda_\gamma}(x) = \Delta(1h^{-1}; 1{,}5h^{-1}; 2h^{-1})$, $\mu_{\lambda_\eta}(x) = \Delta(1h^{-1}; 1{,}5h^{-1}; 2h^{-1})$, $\mu_{\lambda_\psi}(x) = \Delta(1h^{-1}; 1{,}5h^{-1}; 2h^{-1})$, $\mu_{\lambda_\beta}(x) = \Delta(0{,}1h^{-1}; 0{,}15h^{-1}; 0{,}2h^{-1})$, T=500 h, δ=0,1 h, C_0=10000 rub/h, C_1=1000 rub/h, C_2=1500 rub/h, C_3=2000 rub/h, C_4=2200 rub/h, C_5=2500 rub/h, C_β=10000 rub/h. Here Δ designates triangular membership function. Then

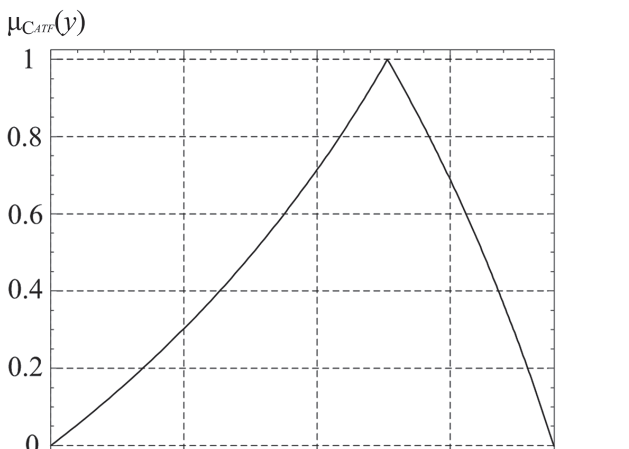

Fig. 1. Membership function of average profit from facility operation in unit of time

Here $M\omega$ received by classical mode is equal 7876 rub/h, which coincides with maximum of membership function and $E[\omega]$ received as a result of defuzzification is 7866 rub/h, which reflects asymmetric property of membership function.

3. Conclusion

This paper proposes the approach to estimation of indicators of economic efficiency of facility "protection object – safety system" taking into account uncertainty of presentation of parameters of the facility. The considered approach is based on the use of concept of random fuzzy values, measure of probability, operator of average of distribution based on Choquet integral, as well as numerical method of evaluations with fuzzy values. Relations are given allowing receipt of membership functions for economic efficiency indicators knowing membership functions of parameters of facility, as well as procedure of defuzzification of the received results is considered.

References

1. **Pereguda A.I., Timashov D.A**. Modeling of process of functioning of APF "PO-SS" with periodically controllable safety system//Reliability. – 2007. – No2. – p. 38-48.

2. **Pereguda A.I**. Mathematical model of reliability of facility «protection object – safety system» at fuzzy initial information//Reliability. – 2014. – No1.-p. 99-113.

3. **Pytiev Y.P.** Possibility as alternative of probability. Mathematical and empirical bases, application. – M: FIZMATLIT, 2007 – 464 p.

4. **Dubois D., Prade H.** The mean value of a fuzzy number // Fuzzy Sets and Systems. – 1987. – Vol. 24. – Pp. 279-300.

5. **Murofushi T., Sugeno M.** An interpretation of fuzzy measures and the choquet integral as an integral with respect to a fuzzy measure // Fuzzy Sets and Systems. – 1989. – Vol. 29. – Pp. 201-227.

6. **Liu B**. Uncertainty Theory. – 2nd edition. – Berlin: Springer-Verlag, 2007. – 255 pp.

7. **Li X., Liu B.** New independence definition of fuzzy random variable and random fuzzy variable // World Journal of Modelling and Simulation. – 2006. – Vol. 2, no. 5. – Pp. 338-342.

8. **Guo R., Zhao R.Q., Guo D., Dunne T.** Random Fuzzy Variable Modeling on Repairable System // Journal of Uncertain Systems. – 2007. – Vol. 1 – Pp. 222-234.

9. **Kwakernaak H.** Fuzzy random variables – I. Definitions and theorems // Information Sciences. – 1978. – Vol. 15 – Pp. 1-29.

10. **Shen Q., Zhao R., Tang W.** Random fuzzy alternating renewal processes // Soft Computing. – 2008. – Vol. 13, no. 2. – Pp. 139-147.

11. **Baichelt F., Franken P.** Reliability and maintenance. Mathematical approach: Translated from German – M.: Radio and communication, 1988. – 392 p.

12. **Hanss M.** Applied Fuzzy Arithmetic: An Introduction with Engineering Applications. – Springer-Verlag, 2005. – 256 pp.

13. **Buckley J.J.** Fuzzy Probability and Statistics. – Springer-Verlag, 2006. – 270 p