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RANKING OF SYSTEM ELEMENTS ON THE BASIS OF FUZZY RELATIONS: THE LEAST INFLUENCE METHOD

This paper proposes a new method of ranking of elements to ensure the reliability of systems with application of the fuzzy relations theory. The ranking problem is formulated as automatic classification on the basis of the transitive closure of similarity fuzzy relation. It makes possible to divide a set of system's elements into disjoint classes undistinguishable by importance.

For construction of a similarity fuzzy relation, each element of a system is represented in the form of a vector of influences. A measure of similarity of pair of elements is the distance between two vectors. Degree of influence of each element is proposed to be calculated by a method of the least influence, which uses expert knowledge about the least influence of an element and comparison of other influences with it by the 9-point Saaty scale.

The proposed method is free from assumption on independence of elements and binary character of reliability: «there is a fault – there is no fault». Possible fields of application of the proposed method are systems with ill-defined structure and multifunctional elements, e.g. organizational, ergatic, military ones, etc.

Keywords: *system, reliability, importance of element, fuzzy relation of influence, transitive closure, cluster analysis.*

1. Introduction

At early stages of system design there is a necessity of evaluation of ranks of its elements. The rank is a quantitative characteristic of element's importance, which is used for solution of the following tasks:

- Development of requirements for reliability of elements on the basis of specified requirements for the reliability of a system as a whole;
- Distribution of resources to increase system's reliability between its elements.

The classical approach for evaluation of ranks uses sensitivity of function of system's reliability to changes of the reliability of its elements. An alternative to this approach are expert opinions formalized by means of fuzzy mathematics.

A widely known example of expert ranking of elements is a popular expression of V.I. Lenin "about post office and telegraph" from his work «Advices of an outsider», which became facetious/ironical "… that it was necessary by all means to capture post office, telegraph, railway stations …" (according to Lenin, these are the first and obligatory conditions of a successful revolt).

The idea of writing this paper appeared during discussions over the report of professor I.B. Herzbach (Karmiel, Israel, 2013), in which ranks of elements were used for distribution of resources aimed at the increase of reliability of roads and buildings in a seismic hazardous area.

In this paper approaches to ranking of system's elements are analyzed and a method of evaluation of ranks on the basis of the theory of fuzzy relations is proposed. It is supposed that importance of an element is defined by its influence on other elements: the more the influence is, the more the importance is.

Initial information on system's structure is formalized in the form of *fuzzy relation of influence,* which is transformed into *fuzzy relation of similarity* and *its transitive closure*. It makes possible to divide a set of system's elements into classes, equivalent in importance.

For calculation of the degrees of influence of system's element on other elements, a special method is proposed using information on the least influence and comparison with it by the 9-point Saaty scale. From here follows the title: *method of least influence*.

In section 2, approaches to ranking of system's elements are analyzed on the basis of cause-and-effect relations between failures.

In section 3, fuzzy relation of influence is introduced as a model of system's structure.

In section 4, method of least influence for formalization of expert knowledge of system is proposed.

In section 5, transition from fuzzy relation of influence to fuzzy relation of similarity is defined.

In section 6, equations for transitive closure of fuzzy relation of similarity and its α-levels are given making it possible to expose classes of elements equivalent in importance.

In section 7, an example illustrating algorithm of application of the proposed method for the system of 5 elements is considered.

2. External and internal approaches

The known approaches to the ranking of system's elements can be divided into two classes on the basis of the type of cause-and-effect relations between failures. We would call these approaches external and internal (Table 1).

Table 1. Approaches to the ranking of elements

Approach	Causes	Consequences
External	Failure of the i-th element	Failure of the system
Internal	Failure of the i- th element	Failure of the j-th element

2.1. External approach

This approach goes back to paper [1] and uses sensitivity of system's reliability to changes of the reliability of its elements (see also [2]). Let us consider the reliability function

$$P_s = f(P_1, P_2, \dots, P_n), \tag{1}$$

which connects probabilities of non-failure system operation (P_S) and its elements (P_i). Let us present this function in the form of a series:

$$P_S = b_0 + \sum_{i=1}^{n} b_i P_i + \sum_{i,j=1}^{n} b_{ij} P_i P_j + \dots, \tag{2}$$

which factors are partial derivatives:

$$b_i = \frac{\partial P_S}{\partial P_i},\ b_{ij} = \frac{\partial^2 P_S}{\partial P_i \partial P_j}. \tag{3}$$

Factor b_i in (3) corresponds to the index of importance of the i-th element (reliability importance index) introduced in paper [1]. Factor b_{ij} in (3) corresponds to index of importance of joint influence of the i-th and j-th elements (joint reliability importance) introduced in paper [3].

For systems, whose reliability is modeled by a Monte-Carlo method, indices of importance (3) are calculated in papers [4, 5].

Paper [6] contains the analysis of the method of evaluation of the system's elements importance directly on the basis of logic (or structural [2]) function

$$\alpha_s = f_L(\alpha_1, \alpha_2, \dots, \alpha_n), \tag{4}$$

where $\alpha_S(\alpha_i) = 1(0)$, , if the system (the i-th element) works (fails), f_L is the Boolean function.

Limitations of the considered group of methods (external approach) consist in the following:

1. Binary character of reliability model (1 – no failure, 0 – failure) does not make it possible to consider intermediate conditions of elements, which only reduce system's efficiency, but do not lead to its complete failure.
2. Supposition about independence of elements does not make it possible to consider the influence of violations in the work of elements on each other.
3. Adequacy of the reliability model (1), on which basis indices of importance (3) are calculated, strongly depends on qualification of an expert writing down a structural function (4). It generates a contradiction between the severity of a differentiation operation (3) and the subjectivity of a model (1), to which this operation is applied. As a result, the received indices of importance of elements cannot possess the property of robustness: they are too sensitive to changes in the structure and parameters of a model (1). In general, L. Zadeh formulated such contradictions as *an incompatibility principle* of high complexity and high accuracy [7]. With reference to indices of importance (3), it means that with increase of system's complexity and uncertainty aspiration for accuracy of calculations loses sense. Here it is pertinent to remind a known aphorism: "mathematicians do everything the right way, but only the thing which is possible to do".

2.2. Internal approach

This approach goes back to evaluation of importance of the elements on the basis of theory of relations and graphs. Transfer of the relations theory into the theory of reliability for the first time was made by V.I. Nechiporenko [8, 9].

The internal approach does not require construction of a structural function (4) and the function of reliability (1).

It relies on the information about a system's structure, i.e. the structure of its elements and links between them. In the meantime, knowledge of influence of violations in some elements on the origination of violations in other elements can be used. For example, "a drift of parameters of the *i*-th element leads to a drift of parameters of the *j*-th element, which leads to a failure of the *k*-th element, etc." Thus, the "domino effect" can be considered.

The constraint matrix serves as a data carrier for evaluation of ranks in [8,9]

$$A = [\alpha_{ij}],\ i,j = 1, 2, \ldots, n, \tag{5}$$

in which $\alpha_{ij} = 1$ (0), if the *i*-th element is connected (not connected) with the *j*-th element.

The rank of the *i*-th element is calculated as the sum of elements of the *i*-th row of a matrix

$$D = A + A^2, \tag{6}$$

which considers one-step and two-step influences of violations in the *i-th* system's element.

The limitation of approach [8, 9] consists in the binary character of the matrix (5), which does not make it possible to consider the force of connections (or influences) between elements. Therefore, there is an interest for generalization of this approach for the case of fuzzy relations [7]. It is necessary to note that the correlation (6) by its structure reminds of a relation of transitive closure, which is used in cluster analysis [10]. It suggests that the task of ranking of elements can be formulated as a problem of automatic classification, which consists in division of a set of elements in classes, equivalent by importance.

Below there is a proposed solution of the problem of ranking of elements on the basis of fuzzy transitive closure [11, 12] and special procedures of construction of fuzzy relations of influence and similarity.

3. Fuzzy relation of influence

Let the $X = \{x_1, x_2, \ldots, x_n\}$ be a set of system's elements. Let us set the influence of the element $x_i \in X$ on other elements by a fuzzy set:

$$I_i = \left\{ \frac{\mu_{i1}}{x_1}, \frac{\mu_{i2}}{x_2}, \ldots, \frac{\mu_{in}}{x_n} \right\}, \tag{7}$$

where μ_{ij} is the number in the range of [0,1] which characterizes degree of influence of element $x_i \in X$ on element $x_j \in X$; $i, j = 1, 2, \ldots, n$.

Let us assume that the influence of element $x_i \in X$ on itself is absent, i.e.

$$\mu_{ii} = 0,\ i = 1, 2, \ldots, n. \tag{8}$$

Collection of degrees of influence μ_{ij} from (7) for all elements $x_i \in X$ forms *fuzzy relation of influence I* specified on a Cartesian product $X \times X$, i.e. $I \subset X \times X$:

$$I = \left[\frac{\mu_{ij}}{(x_i, x_j)} \right],\ i, j = 1, 2, \ldots, n. \tag{9}$$

Number μ_{ij}, which is put in correspondence to each pair of elements (x_i, x_j), can be set by experts or by a method *of least influence,* which is proposed below. Let us note that similar procedure of calculation of membership degrees was used earlier in the paper [13].

4. Method of least influence

Let f_{ij} be the force of influence of element $x_i \in X$ on element $x_j \in X$, and the condition is satisfied: "the more is the force f_{ij}, the more is the degree of influence μ_{ij}", i.e. the correlation takes place:

$$\frac{\mu_{i1}}{f_{i1}} = \frac{\mu_{i1}}{f_{i1}} = \ldots = \frac{\mu_{il}}{f_{il}} = \ldots = \frac{\mu_{in}}{f_{in}}. \tag{10}$$

It is supposed that according to (8):

$$f_{ii} = 0,\ i = 1, 2, \ldots, n. \tag{11}$$

Let x_l be an element, on which the element has the least influence. From (10) we have:

$$\mu_{i1} = \mu_{il} \frac{f_{i1}}{f_{il}},\ \mu_{i2} = \mu_{il} \frac{f_{i2}}{f_{il}}, \ldots, \mu_{in} = \mu_{il} \frac{f_{in}}{f_{il}}. \tag{12}$$

Substituting (12) in the requirement

$$\mu_{i1} + \mu_{i2} + \ldots + \mu_{in} = 1,\ i = 1, 2, \ldots, n,$$

we receive the least degree of influence of element $x_i \in X$ in the system:

$$\mu_{il} = \left(\frac{f_{i1}}{f_{il}} + \frac{f_{i2}}{f_{il}} + \ldots + \frac{f_{in}}{f_{il}} \right)^{-1}. \tag{13}$$

Correlations (13) and (12) make it possible to calculate influence degrees in fuzzy relation (9) by comparison of forces of influences f_{ij} with the least force of influence f_{il} for each element $x_i \in X$. The 9-point Saaty scale [14] is used for this purpose [14]

$$\frac{f_{ij}}{f_{il}} = 1, 3, 5, 7, 9, \tag{14}$$

if influence «*ij*» (element x_i on element x_j) in comparison with the least influence «*i ℓ*» (element x_i on element x_l): 1 – the same, 3 – little more, 5 – more, 7 – much more, 9 – absolutely more (intermediate estimations are possible: 2, 4, 6, 8).

5. Fuzzy relation of similarity

The measure of similarity by degree of influence between elements $x_i \in X$ and $x_j \in X$ shall be defined by the value

$$r_{ij} = 1 - d_{ij}, \quad (15)$$

where d_{ij} is a distance between fuzzy sets of influence of elements x_i and x_j:

$$I_i = \left\{ \frac{\mu_{i1}}{x_1}, \frac{\mu_{i2}}{x_2}, ..., \frac{\mu_{in}}{x_n} \right\},$$

$$I_j = \left\{ \frac{\mu_{j1}}{x_1}, \frac{\mu_{j2}}{x_2}, ..., \frac{\mu_{jn}}{x_n} \right\}.$$

For calculation d_{ij}, relative distances as per Hamming ($d_{ij}^{(h)}$) or as per Euclid ($d_{ij}^{(e)}$) can be used:

$$d_{ij}^{(h)} = \frac{1}{n} \sum_{k=1}^{n} \left| \mu_{ik} - \mu_{jk} \right|, \quad (16)$$

$$d_{ij}^{(h)} = \frac{1}{n} \sqrt{\sum_{k=1}^{n} \left(\mu_{ik} - \mu_{jk} \right)^2}, \quad (17)$$

Collection of values r_{ij} for all pairs $(x_i, x_j) \in X \times X$ forms fuzzy relation of similarity $R \subset X \times X$:

$$R = [r_{ij} / (x_i, x_j)], \quad (18)$$

which possesses the following properties:
(a) *reflexivity,* i.e. $r_{ij} = 1$, for all $x_i \in X$,
(b) *symmetry*, i.e. $r_{ij} = r_{ji}$, for all $x_i, x_j \in X$.

6. Classification and ranking of elements

For dividing a set X into non-intersected classes of elements similar by degree of influence, it is necessary to give to the initial intransitive relation of similarity R the property of transitivity. Such transformation is ensured by operation of a transitive closure of fuzzy relation, for the first time considered in [11, 12].

Transitive closure of relation R is the relation $\hat{R}$ defined as follows:

$$\hat{R} = R^1 \cup R^2 \cup ... \cup R^k \cup ..., \quad (19)$$

where relations R^k are defined recursively:

$$R^1 = R, \ R^k = R^{k-1} \circ R, \ k = 2, 3, ..., n;$$

$\cup$ – Operation of combination of fuzzy relations;
$\circ$ – Operation of a fuzzy composition.
Operations on matrices of relations are carried out according to the scheme:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cup \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \vee e \,|\, b \vee f \\ c \vee e \,|\, d \vee h \end{bmatrix},$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \circ \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ((a \wedge e) \vee (b \wedge g)) \,|\, ((a \wedge f) \vee (b \wedge h)) \\ ((c \wedge e) \vee (d \wedge g)) \,|\, ((c \wedge f) \vee (d \wedge h)) \end{bmatrix},$$

$$\wedge = \min, \ \vee = \max.$$

This computational scheme, which is exemplified by the matrix 2×2, remains the same for matrices of arbitrary dimension.

It is natural to assume that the rank of the element x_i depends on the quantity of its links with other system's elements similar by degree of influence, $i = 1, 2, …, n$. Therefore, considering transitive connections, the element's rank will be defined as the sum of elements of the i-th row of $\hat{R}$ matrix (19).

Classes of elements similar by degree of influence are organized by expansion of relation $\hat{R}$ on α-levels (sections):

$$\hat{R} = \bigcup_{\alpha} \alpha \hat{R}_{\alpha}, \ \alpha \in [0,1], \quad (20)$$

where $\hat{R}_{\alpha}$ is α-level of relation $\hat{R}$.
For a fuzzy relation $\hat{R}$ (19) written in the form

$$\hat{R} = \left[\frac{\hat{r}_{ij}}{(x_i, x_j)} \right], \ (x_i, x_j) \in X \times X, \ i, j = 1, 2, ..., n,$$

a common relation $\hat{R}_{\alpha}$ consists of the pairs (x_i, x_j), whose membership degree is not less than α, i.e.

$$\hat{R} = \left[\frac{1(0)}{(x_i, x_j)} \right], \text{ здесь } r_{ij} \geq \alpha \ (r_{ij} < \alpha). \quad (21)$$

7. Example

Let us consider the system of 5 elements, i.e. $X = \{x_1, x_2, x_3, x_4, x_5\}$. Expert information necessary for calculation of relation (9) by the method of least influence is presented in Table 2.

Table 2. Initial data for the least influence method

x_i	x_l	ij / il				
		x_1	x_2	x_3	x_4	x_5
x_1	x_2					
x_2	x_1					
x_3	x_5					
x_4	x_2					
x_5	x_1					

The second column of Table 2 (x_l) contains elements, on which corresponding elements of the first column (x_i) have the least influence: x_1 influences slightest on x_2, x_2 influences slightest on x_1, …, x_5 influences slightest on x_1. An expert is a source of this information.

Cells of Table 2 contain expert comparisons of influence forces f_{ij} with the smallest forces of influence f_{il}. For brevity, instead of f_{ij}/f_{il} it is written *ij*/*il*:

- the notation shows that the influence of element x_1 on element x_3 *is more* (5) than the influence of x_1 on x_2;

- the notation shows that the influence of element x_5 on element x_3 *is absolutely more* (9) than the influence of x_5 on x_1, etc.

Zero values in cells of Table 2 correspond to the fact that $f_{ii} = 0$ for all $i = 1, 2, \ldots, 5$.

Using data from Table 2 and equations (13) and (12), we will calculate degrees of influence μ_{ij} for correlation (9). Here $\mu_{ii} = 0$.

For element x_1 ($i = 1$, $\ell = 2$) we have:

$$\mu_{11} = 0,$$

$$\mu_{12} = \left(\frac{f_{11}}{f_{12}} + \frac{f_{12}}{f_{12}} + \frac{f_{13}}{f_{12}} + \frac{f_{14}}{f_{12}} + \frac{f_{15}}{f_{12}}\right)^{-1} = \frac{1}{0+1+5+3+1} = \frac{1}{10},$$

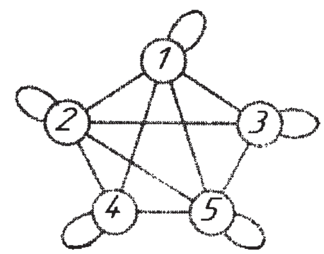
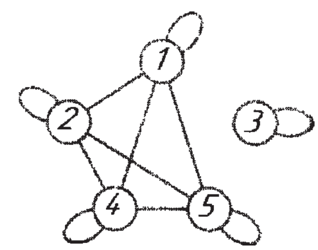
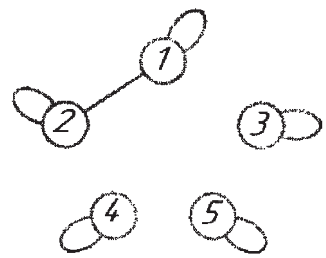
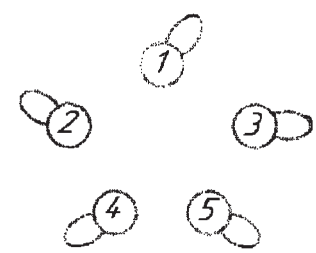
$$\mu_{13} = \mu_{12}\frac{f_{13}}{f_{12}} = \frac{1}{10}\cdot 5 = \frac{5}{10},$$

$$\mu_{14} = \mu_{12}\frac{f_{14}}{f_{12}} = \frac{1}{10}\cdot 3 = \frac{3}{10},$$

$$\mu_{15} = \mu_{12}\frac{f_{15}}{f_{12}} = \frac{1}{10}\cdot 1 = \frac{1}{10}.$$

Remaining membership degrees, which form a fuzzy influence relation, are received similarly:

Table 3. Relations of the α-level and their graphs

α	R_α						Graph
		1	2	3	4	5	
0,53	1	1	1	1	1	1	
	2	1	1	1	1	1	
	3	1	1	1	1	1	
	4	1	1	1	1	1	
	5	1	1	1	1	1	
		1	2	3	4	5	
0,83	1	1	1	0	1	1	
	2	1	1	0	1	1	
	3	0	0	1	0	0	
	4	1	1	0	1	1	
	5	1	1	0	1	1	
		1	2	3	4	5	
0,87	1	1	1	0	0	0	
	2	1	1	0	0	0	
	3	0	0	1	0	0	
	4	0	0	0	1	0	
	5	0	0	0	0	1	
		1	2	3	4	5	
1	1	1	0	0	0	0	
	2	0	1	0	0	0	
	3	0	0	1	0	0	
	4	0	0	0	1	0	
	5	0	0	0	0	1	

$$I=\begin{array}{c|ccccc l} & x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline x_1 & 0 & 1/10 & 5/10 & 3/10 & 1/10 & \max=5/10 \\ x_2 & 1/14 & 0 & 9/14 & 3/14 & 1/14 & \max=9/14 \\ x_3 & 9/15 & 3/15 & 0 & 2/15 & 1/15 & \max=9/15 \\ x_4 & 1/12 & 1/12 & 7/12 & 0 & 3/12 & \max=7/12 \\ x_5 & 1/22 & 5/22 & 9/22 & 7/22 & 0 & \max=9/22 \end{array} \quad (22)$$

For normalization of correlation (22) we will divide the elements of every line by maximum value and we will receive:

$$I=\begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 0 & 0,2 & 1 & 0,6 & 0,2 \\ x_2 & 0,11 & 0 & 1 & 0,33 & 0,11 \\ x_3 & 1 & 0,33 & 0 & 0,22 & 0,11 \\ x_4 & 0,14 & 0,14 & 1 & 0 & 0,43 \\ x_5 & 0,11 & 0,56 & 1 & 0,78 & 0 \end{array} \quad (23)$$

The fuzzy similarity relation (18) received from (23) looks like:

$$R=\begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 1 & 0,87 & 0,48 & 0,76 & 0,83 \\ x_2 & 0,87 & 1 & 0,53 & 0,83 & 0,44 \\ x_3 & 0,48 & 0,53 & 1 & 0,48 & 0,44 \\ x_4 & 0,79 & 0,83 & 0,48 & 1 & 0,67 \\ x_5 & 0,83 & 0,78 & 0,44 & 0,67 & 1 \end{array} \quad (24)$$

Membership degrees in (24) are received from (23) with use of distance by Hamming (16). For example, $r_{12} = 1 - d_{12}$, where

$$d_{12}=\frac{1}{5}\left[(0,\ 0,2,\ 1,\ 0,6,\ 0,2)-(0,11,\ 0,\ 1,\ 0,33,\ 0,1)\right]=$$
$$=\frac{1}{5}\left[|0-0,11|+|0,2-0|+|1-1|+|0,6-0,33|+|0,2-0,1|\right]=$$
$$=\frac{1}{5}\left[0,11+0,1+0+0,27+0,09\right]=0,13$$

For deriving transitive closure of similarity relation (19) from correlation (24) we find:

$$R^2=R\circ R=\begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 1 & 0,87 & 0,53 & 0,83 & 0,83 \\ x_2 & 0,87 & 1 & 0,53 & 0,83 & 0,83 \\ x_3 & 0,53 & 0,53 & 1 & 0,53 & 0,53 \\ x_4 & 0,83 & 0,83 & 0,53 & 1 & 0,79 \\ x_5 & 0,83 & 0,83 & 0,53 & 0,79 & 1 \end{array},$$

$$R^3=R^2\circ R=$$
$$=\begin{array}{c|ccccc} & x_1 & x_2 & x_3 & x_4 & x_5 \\ \hline x_1 & 1 & 0,87 & 0,53 & 0,83 & 0,83 \\ x_2 & 0,87 & 1 & 0,53 & 0,83 & 0,83 \\ x_3 & 0,53 & 0,53 & 1 & 0,53 & 0,53 \\ x_4 & 0,83 & 0,83 & 0,53 & 1 & 0,83 \\ x_5 & 0,83 & 0,83 & 0,53 & 0,83 & 1 \end{array}= \quad (25)$$
$$=R^4=R^5=\ldots=R^\infty.$$

Therefore, transitive closure (19) in our case looks like:

$$\hat{R}=R\cup R^2\cup R^3\cup\ldots\cup R^k\cup\ldots=R^3, \quad (26)$$

i.e. coincides with correlation (25).

Summarizing values of rows of matrix (25), we receive quantitative values of ranks of elements:

$$\rho_1 = 1 + 0,87 + 0,53 + 0,83 = 4,06,$$
$$\rho_2 = 4,06,\ \rho_3 = 3,12,\ \rho_4 = 4,02,\ \rho_5 = 4,02. \quad (27)$$

Fuzzy relation (26) can be factorized by α-levels as follows:

$$\hat{R}=\bigcup_{\alpha}\alpha R_{\alpha}=0,53R_{0,53}\bigcup 0,83R_{0,83}\bigcup 0,87R_{0,87}\bigcup R_1,$$

where well-defined relations of α-level R_α and their graphs are presented in Table 3. Here for brevity element x_i is designated by numeral i, $i = 1, 2, \ldots, 5$. Well-defined relations of α-level (Table 3) form classes of elements equivalent by importance (Table 4). The tree of decomposition of a set of system's elements in equivalence classes is presented in Fig. 1.

Table 4. Classes of elements equivalent in importance

Level	Number of classes	Classes of elements
α = 0,53	1	$\{x_1, x_2, x_3, x_4, x_5\}$
α = 0,83	2	$\{x_1, x_2, x_4, x_5\}$, $\{x_3\}$
α = 0,87	4	$\{x_1, x_2\}$, $\{x_3\}$, $\{x_4\}$, $\{x_5\}$
α = 1	5	$\{x_1\}$, $\{x_2\}$, $\{x_3\}$, $\{x_4\}$, $\{x_5\}$

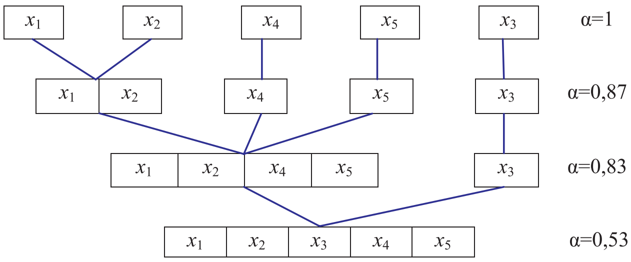


Fig.1. He tree of decomposition of a set of system's elements in equivalence classes

Number α can be interpreted as a level of definiteness of our knowledge of the system, and (1 – α) is uncertainty level. Naturally, the more complicated the system is and the more is the number of realities not considered during modeling, the more is the uncertainty and low is number α.

From Fig. 1 it is evident that at maximum definiteness (α = 1), each of elements xi represents a unique class of importance. However, at level α = 0,53 all system's elements are not distinguishable by ranks. Taking into account quantitative estimations (27) for practical calculations it is possible to choose definiteness level α = 0,83, on which:

$$\rho_1 = \rho_2 = \rho_4 = \rho_5 \approx 4,\ \ \rho_3 \approx 3.$$

If C_0 are admissible expenses on assurance of the system's reliability, taking into account ranks of elements these expenses should be distributed as follows:

$$\sum_{i=1}^{5} C_i = C_0,\ \ C_1 = C_2 = C_4 = C_5 = \frac{4}{19} C_0,\ \ C_3 = \frac{3}{19} C_0.$$

Similarly, if λ_0 is a required system's failure rate, for the exponential law of reliability and the elementary consecutive scheme we would receive the required λ-characteristics of elements:

$$\sum_{i=1}^{5} \lambda_i = \lambda_0,\ \ \lambda_1 = \lambda_2 = \lambda_4 = \lambda_5 = \frac{4}{19} \lambda_0,\ \ \lambda_3 = \frac{3}{19} \lambda_0.$$

8. Conclusion

Ranking of system elements at the design stage, from the point of view of reliability, can be carried out with application of two fundamentally different approaches: external and internal ones.

The external approach uses sensitivity of system's reliability to changes of the reliability of its elements. The internal approach is based on knowledge of the influence of failures of each of elements on failures of other elements.

Within the framework of the internal approach this paper proposes a new method of ranking of elements with application of the fuzzy relations theory. The task of ranking is reduced to automatic classification on the basis of transitive closure of the fuzzy relation of similarity. It makes it possible to divide a set of the system's elements into disjoint classes not undistinguishable by importance.

For construction of the fuzzy relation of similarity each system's element is represented in the form of vector of influences. A measure of similarity of pair of elements is the distance between two vectors. It is proposed to calculate the degree of influence of each element on other elements by the method of the least influence. This method uses expert knowledge of the least influence of an element and comparison of other influences to it by the 9-point Saaty scale.

The proposed method is free from limitations inherent to methods of external approach, which are connected with assumptions on independence of elements and binary character of reliability, viz. "failure – no failure". Possible fields of application of the proposed method are the systems with ill-defined structure and multifunctional elements, e.g. organizational, ergatic, military ones, etc.

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